CSE 40171: Artificial Intelligence

Probability: Intro to Probability 1
Instructions for the group project proposal have been released. It is due at 11:59PM on 10/12.
If you went to Terry Boult’s talk, send your write-up summarizing the talk to Abby (agraese@nd.edu) for extra credit.
Course Roadmap

Introduction
(week 1)

Problem Solving
(weeks 3 - 6)

Machine Learning
(weeks 11 - 16)

Biological Intelligence
(week 2)

Probabilistic Reasoning
(weeks 7 - 10)
Games with no element of chance
Games with some element of chance

Seven Wonders Game © BY 2.0 Schezar
Games of Chance
“The Matrix has its roots in primitive arcade games,’ said the voice-over, 'in early graphics programs and military experimentation with cranial jacks.' On the Sony, a two-dimensional space war faded behind a forest of mathematically generated ferns, demonstrating the spatial possibilities of logarithmic spirals; cold blue military footage burned through, lab animals wired into test systems, helmets feeding into fire control circuits of tanks and war planes. 'Cyberspace. A consensual hallucination experienced daily by billions of legitimate operators, in every nation, by children being taught mathematical concepts... A graphic representation of data abstracted from the banks of every computer in the human system. Unthinkable complexity. Lines of light ranged in the nonspace of the mind, clusters and constellations of data. Like city lights, receding...”
Unstructured Environment
Acting Under Uncertainty

Agents need to handle uncertainty

- Due to partial observability
- Due to non-determinism
- Due to a combination of the two
Problems with Belief States

- Partial sensor information: the agent has to consider every logically possible explanation for the available observations.

- Contingency plans: as the state space grows, so does the space for contingency planning.

- What if we don’t have a plan? We still need to choose an action.
Rationality

**Russell and Norvig tell us:** The right thing to do — the *rational decision* — therefore depends on both the relative importance of various goals and the likelihood that, and the degree to which, they will be achieved.
The Frequentist Philosophy

Numbers can only come from experiments

**Example:** If we test 100 people and find that 10 of them have a cavity, then we can say the probability of a cavity is approximately 0.1.

From any finite sample, we can estimate the true fraction and also calculate how accurate our estimate is likely to be.
Summarizing Uncertainty

Let’s try to diagnose a patient’s toothache:

\[ \text{Toothache} \implies \text{Cavity}. \]

\[ \text{Toothache} \implies \text{Cavity} \lor \text{GumProblem} \lor \text{Abscess} \ldots \]

\[ \text{Cavity} \implies \text{Toothache}. \]
Why does logic fail for medical diagnosis?

• **Laziness:** It is too much work to list the complete set of antecedents or consequences needed to ensure an exceptionalness rule and too hard to use such rules.

• **Theoretical ignorance:** Medical science has no complete theory for the domain.

• **Practical ignorance:** Even if we know all the rules, we might be uncertain about a particular patient because not all the necessary tests have been or can be run.
Probability Theory as an Alternative

Probability provides a way of summarizing the uncertainty that comes from our laziness and ignorance.

We might not know what for sure afflicts a particular patient, but maybe there is an 80% chance that a patient with a toothache has a cavity.

This belief could come from statistical data.
Or it could come from general dental knowledge.
Or it could come from some combo of evidence.
Decision Theory

Decision Theory = Probability Theory + Utility Theory

An agent is rational if and only if it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action.
Reminder: Probabilities

A **random variable** represents an event whose outcome is unknown.

A **probability distribution** is an assignment of weights to outcomes.
Reminder: Probabilities

Example: Traffic on freeway

- Random variable: \( T = \) whether there’s traffic
- Outcomes: \( T \) in \{none, light, heavy\}
- Distribution: \( P(T = \text{none}) = 0.25, \ P(T = \text{light}) = 0.50, \ P(T = \text{heavy}) = 0.25 \)
Reminder: Probabilities

Some laws of probability (more later):

- Probabilities are always non-negative
- Probabilities over all possible outcomes sum to one

As we get more evidence, probabilities may change:

- \( P(T = \text{heavy}) = 0.25, \ P(T = \text{heavy} \ | \ \text{Hour} = 8\text{am}) = 0.60 \)

- We’ll talk about methods for reasoning and updating probabilities later
Sample Space

The set of all possible worlds is called the *sample space*

The possible worlds are mutually exclusive and exhaustive

**Example: rolling two dice**

36 possible worlds: (1,1), (1,2), ..., (6,6)
Probability Model

\[ \Omega = \text{sample space} \quad \omega = \text{elements of the space} \]

\[ 0 \leq P(\omega) \leq 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1 \]

Probability of each possible world

Need to sum to 1
Propositions

Probabilistic assertions and queries are not usually about particular possible worlds, but about sets of them.

We will call these sets events.

The events are always described by propositions in a formal language.

The probability associated with a proposition is defined to be the sum of the probabilities of the worlds in which it holds:

\[ P(\phi) = \sum_{\omega \in \phi} P(\omega) \]
Unconditional or Prior Probabilities

Probabilities such as $P(Total = 11)$ ⚒ ⚒ and $P(doubles)$ ⚒ ⚒ are called unconditional or prior probabilities.

Such probabilities refer to degrees of belief in propositions in the absence of any other information.
Conditional or Posterior Probabilities

Most of the time, we have some evidence that has already been revealed

**Example:** we have rolled two dice. The first die shows a 5, and we are waiting for a result from the second. 🎲?

What is the probability of rolling doubles given the first die is a 5? \( P(\text{doubles} \mid \text{Die}_1 = 5) \)
Conditional or Posterior Probabilities

Conditional probabilities are defined in terms of unconditional probabilities; for any propositions \(a\) and \(b\), we have:

\[
P(a|b) = \frac{P(a \land b)}{P(b)}
\]

Which holds true whenever \(P(b) > 0\)

Simpler form: product rule

\[
P(a \land b) = P(a|b)P(b)
\]
The Language of Propositions

Propositions describing sets of possible worlds are written in a notation that combines elements of propositional logic and constraint satisfaction notation.

Let’s look at a few examples…
Probability Distributions

\[ P(Weather) = \langle 0.6, 0.1, 0.29, 0.01 \rangle \]

A probability distribution for the random variable \( Weather \)

The \( P \) notation can also be used for conditional distributions:

\[ P(X \mid Y) \]

gives the values of \( P(X = x_i \mid Y = y_j) \) for each possible \( i,j \) pair

We can only do this for discrete domains
Joint Probability Distributions

We also need notation for distributions on multiple variables:

\[ P(Weather, Cavity) = \text{4x2 table of probabilities called the joint probability distribution of } Weather \text{ and } Cavity \]

\[ P(sunny, Cavity) = \text{two-element vector} \]

\[ P(Weather, Cavity) = P(Weather \mid Cavity) \ P(Cavity) = \text{product rules for all possible values of } Weather \text{ and } Cavity \]
Probability Axioms

**Example:** derivation for the familiar relationship between the probability of a proposition and the probability of its negation

\[
P(\neg a) = \sum_{\omega \in \neg a} P(\omega)
\]

\[
= \sum_{\omega \in \neg a} P(\omega) + \sum_{\omega \in a} P(\omega) - \sum_{\omega \in a} P(\omega)
\]

\[
= \sum_{\omega \in \Omega} P(\omega) - \sum_{\omega \in a} P(\omega)
\]

\[
= 1 - P(a)
\]
Inclusion-Exclusion Principle

The probability of disjunction is also useful:

\[ P(a \lor b) = P(a) + P(b) - P(a \land b) \]
Kolmogorov’s Axioms

\[ 0 \leq P(\omega) \leq 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1 \]

\[ P(a \lor b) = P(a) + P(b) - P(a \land b) \]

Why can’t an agent hold the following set of beliefs?

\[ P(a) = 0.4 \quad P(a \land b) = 0.0 \]

\[ P(b) = 0.3 \quad P(a \lor b) = 0.8 \]