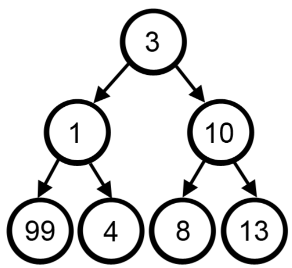
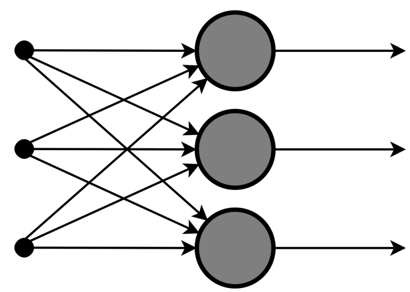
CSE 40171: Artificial Intelligence





Constraint Satisfaction Problems: Inference and Backtracking Search

Homework #3 has been released
It is due at 11:59PM on 10/9

In the regular state-space search algorithms, we could only do one thing:

Search

With CSPs, we can do two things:

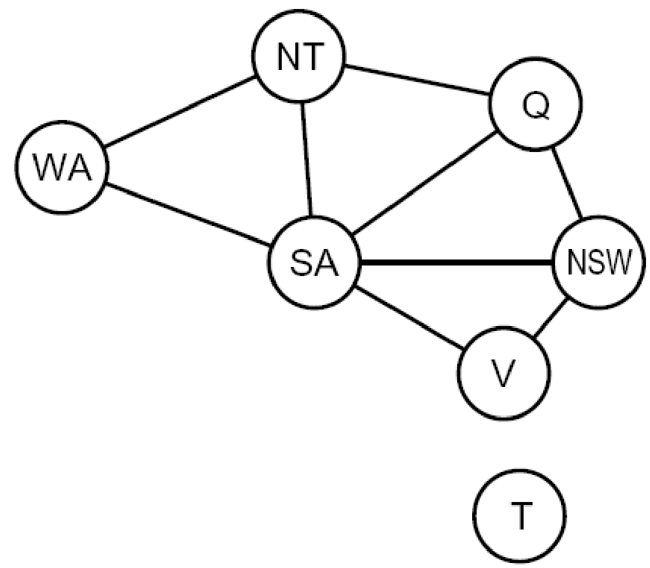
1. **Search**
2. **Use constraints to reduce the number of legal values for a variable**

↑

And this reduction can propagate to neighboring variables

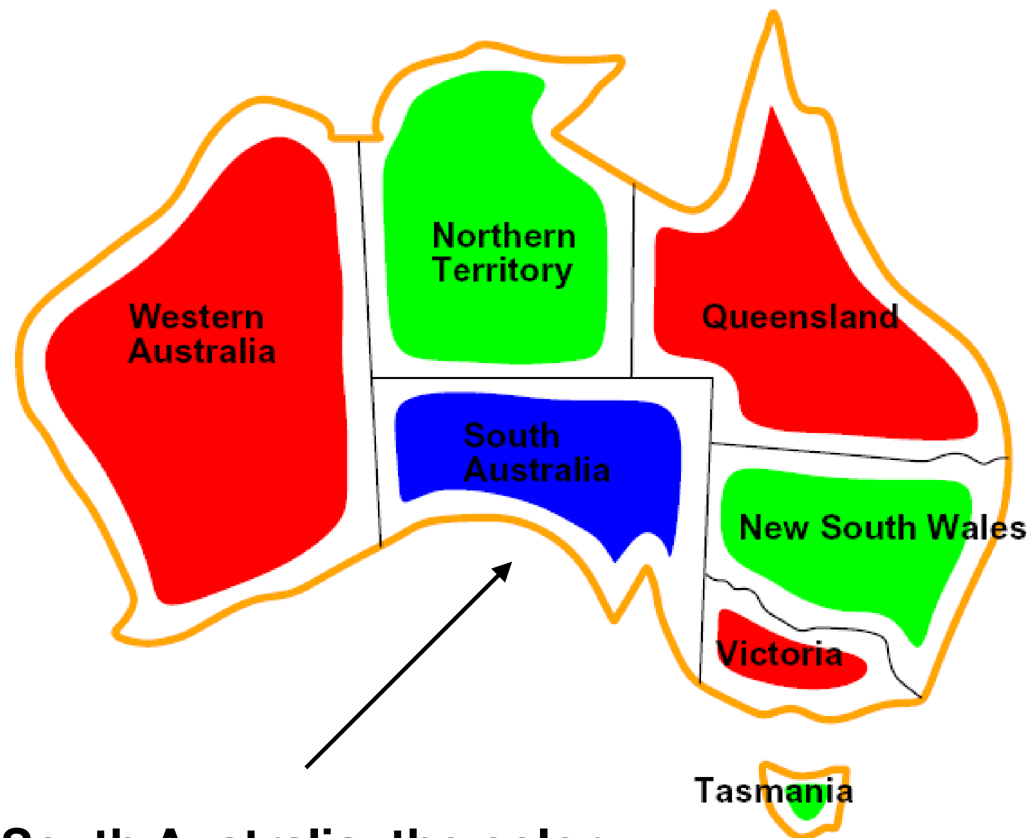
Local Consistency

Enforcing local consistency in each part of the graph causes inconsistent values to be eliminated throughout the graph



So *where* do we enforce it?

Vertex Consistency

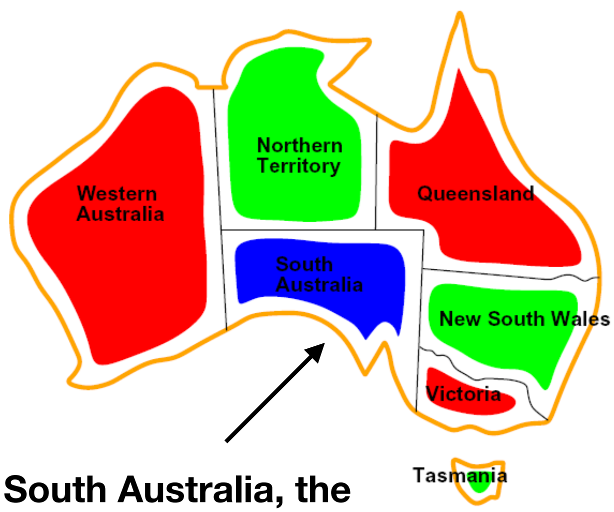


In South Australia, the color green is disliked

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Vertex Consistency

A single variable is **vertex-consistent** if all the values in its domain stratify its unary constraints



In South Australia, the color green is disliked

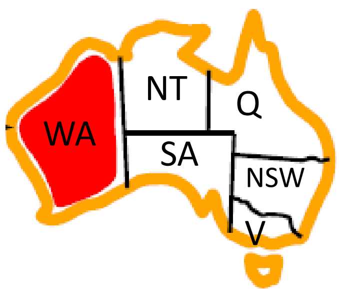
Starting Domain: $\{red, green, blue\}$

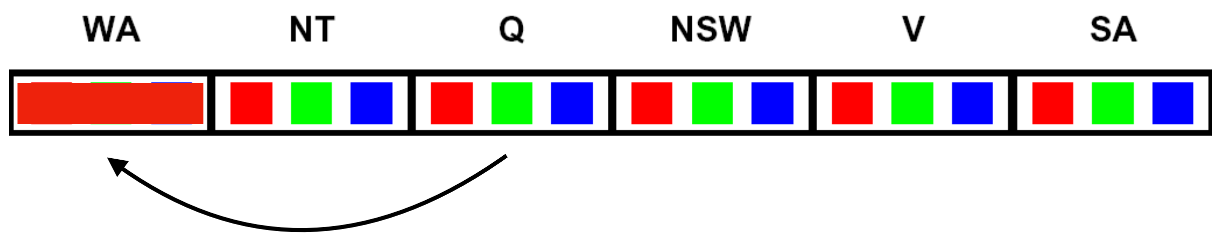
Reduced Domain: $\{red, blue\}$

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Edge Consistency

An edge $X \rightarrow Y$ is **consistent** iff for every x in the tail there is some y in the head which could be assigned without violating a constraint



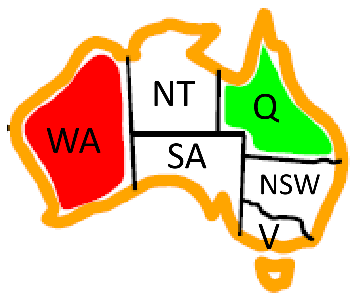


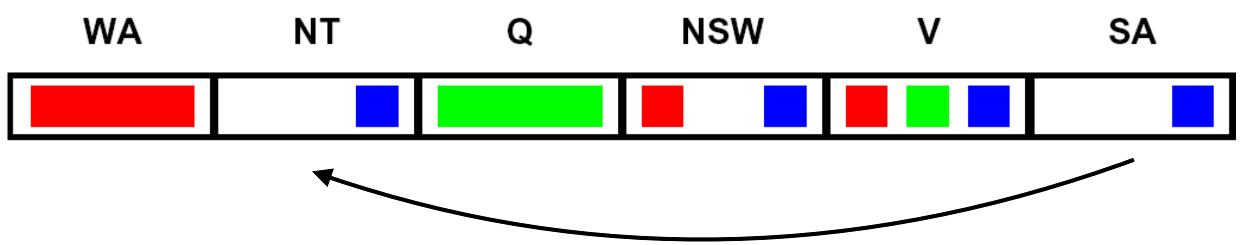
Forward Checking: Enforcing consistency of edges pointing to each new assignment

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Edge Consistency of an Entire CSP

A simple form of propagation makes sure **all** edges are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked
- Edge consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing edge consistency?

Remember: Delete from the tail!

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Enforcing Edge Consistency in a CSP

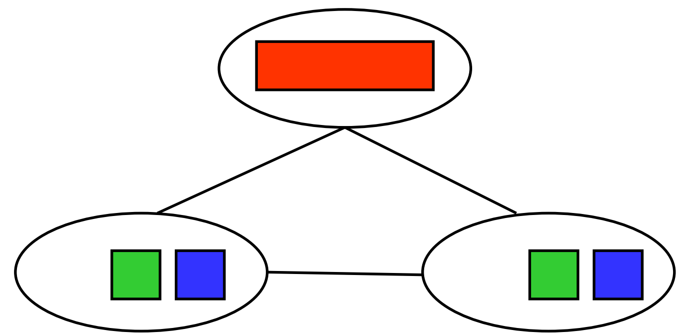
```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_n}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (X_i, X_j) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
         for each X_k in NEIGHBORS[X_i] do
            add (X_k, X_i) to queue
```

```
function REMOVE-INCONSISTENT-VALUES( X_i, X_j) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[X_i] do
      if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint X_i ↔ X_j
         then delete x from DOMAIN[X_i];  removed ← true
   return removed
```

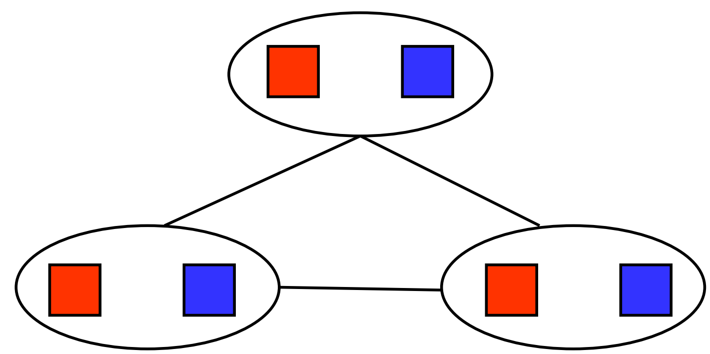
- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- …but detecting all possible future problems is NP-hard – why?

Limitations of Edge Consistency



After enforcing arc consistency:

- Can have one solution left
- Can have multiple solutions left
- Can have no solutions left (and not know it)

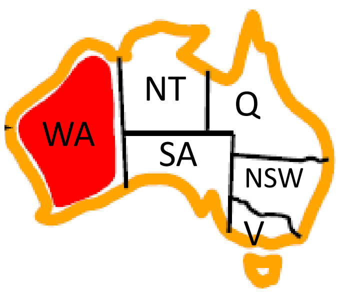


What went wrong here?

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Path Consistency

Tightens the binary constraints by using implicit constraints that are inferred by looking at triples of variables



Let's color the map with two colors:

Make the set $\{WA, SA\}$ path consistent with respect to NT

Two options: $\{WA = red, SA = blue\}$ and $\{WA = blue, SA = red\}$

What do we assign to NT?

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

K-consistency

Let's generalize the notion of consistency:

A CSP is K-consistent if, for any set of $k-1$ variables, and for any consistent assignment to those variables, a consistent value can always be assigned to any kth variable.

1-consistency?	vertex consistency
2-consistency?	edge consistency
3-consistency?	path consistency

K-consistency

A CSP is strongly K-consistent if it is also $(k-1)$-consistent, $(k-2)$-consistent, all the way down to 1-consistent.

Assume we have a CSP with n nodes and want to make it strongly n-consistent:

> For each variable X_i, we only need to search through the d values in the domain to find a value consistent with $X_1, \ldots, X_{i-1}$.
>
> Guaranteed solution in time $O(n^2d)$

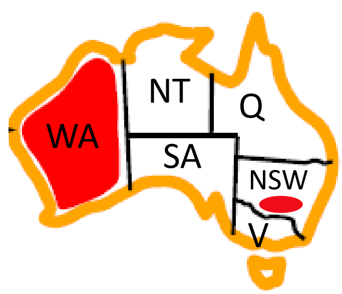
time is exponential in n in the worst case!

space is also exponential in n!

Global Constraints

The *Alldiff* constraint states that all variables involved must have distinct values

Inconsistency detector: if m variables are involved in the constraint, and if they have n possible district values, and $m > n$, then the constraint cannot be satisfied



Can we detect the inconsistency in the assignment $\{WA = red, NSW = red\}$?

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Resource Constraints

We can also have an *Atmost* constraint, meaning no more than n resources can be assigned

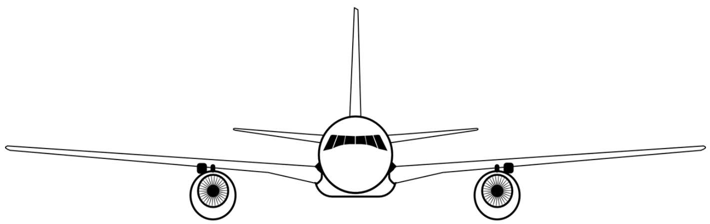
Scheduling Example:

# of personnel assigned to 4 tasks: $P_1, \ldots, P_4$

Constraint that no more than 10 people are assigned:
$Atmost\ (10, P_1, P_2, P_3, P_4)$

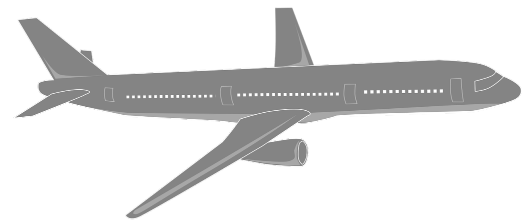
If the domain of each variable is ~~{3, 4, 5, 6}~~, is *Atmost* satisfied?

Bounds Propagation



Capacity: 165

$D_1 = [0, 165]$



Capacity: 385

$D_2 = [0, 385]$

Additional Constraint: the two flights must carry 420 people

$D_1 = [35, 165]$

$D_2 = [255, 385]$

Backtracking Search

Backtracking Search

Some problems, like Sudoku, can be solved by inference over constraints

- But this is not true for all problems

Backtracking search is the basic uninformed algorithm for solving CSPs

Idea 1: One variable at a time

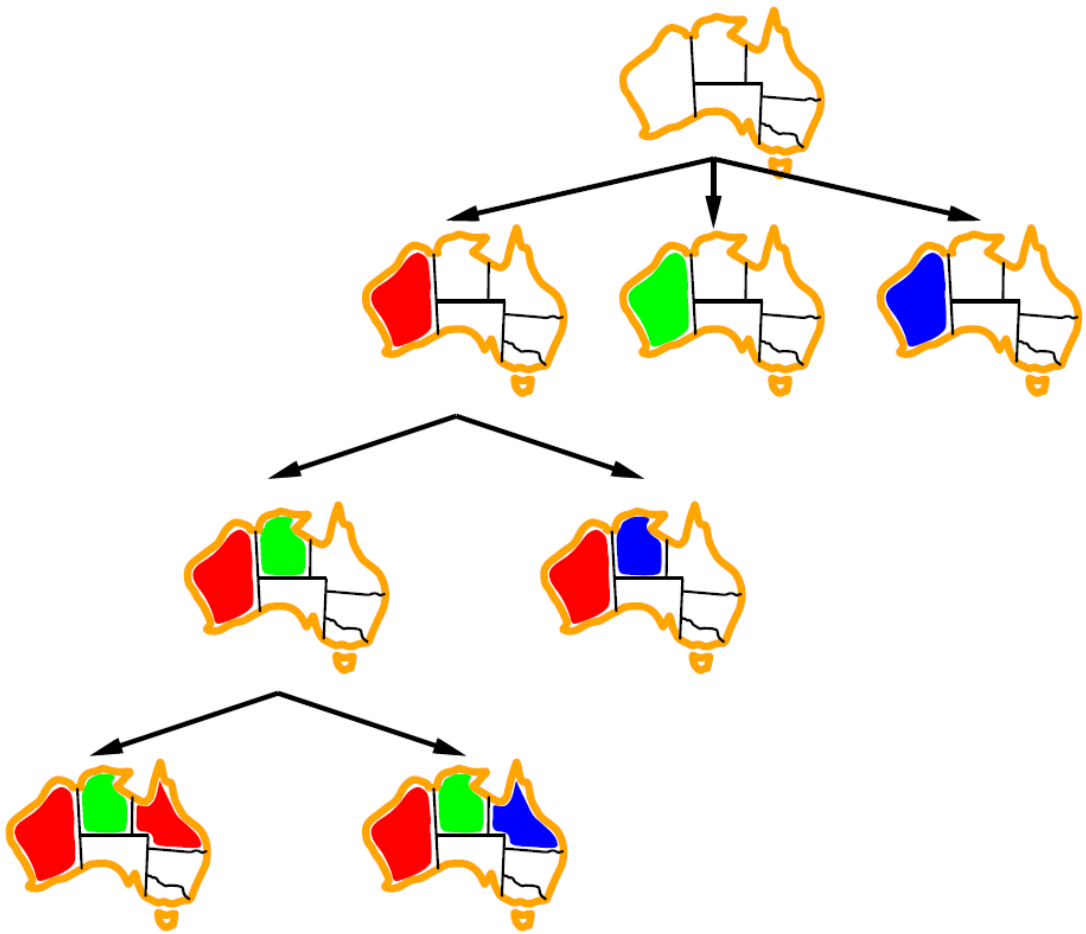
- Variable assignments are commutative, so fix ordering
- For example, [$WA = red$ then $NT = green$] is the same as [$NT = green$ then $WA = red$]
- Only need to consider assignments to a single variable at each step

Idea 2: Check constraints as you go

- For example, consider only values which do not conflict with previous assignments
- Might have to do some computation to check the constraints
- “Incremental goal test”

Slide credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Backtracking Example



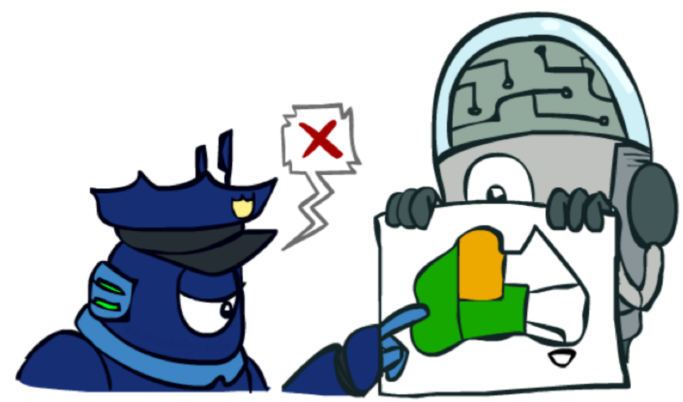


Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Backtracking Search

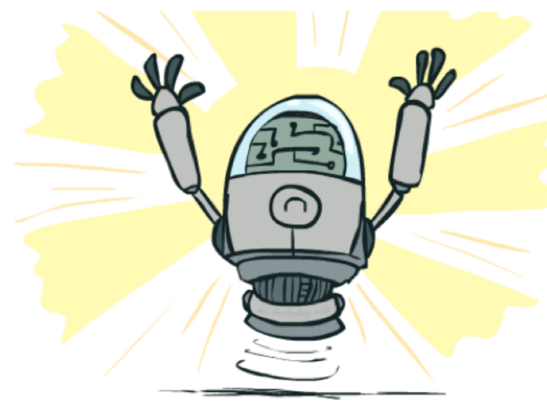
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
  return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
  for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
     if value is consistent with assignment given CONSTRAINTS[csp] then
        add {var = value} to assignment
        result ← RECURSIVE-BACKTRACKING(assignment, csp)
        if result ≠ failure then return result
        remove {var = value} from assignment
  return failure
```

Backtracking = DFS + variable-ordering + fail-on-violation

Improving Backtracking Search

- General-purpose ideas give huge gains in speed
- Filtering: Can we detect inevitable failure early?
- Ordering
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Structure: Can we exploit the problem structure?



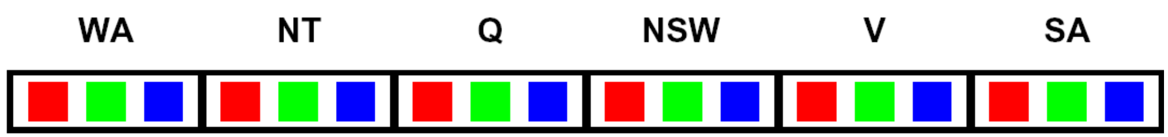
Filtering

Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Filtering: Forward Checking

Filtering: Keep track of domains for unassigned variables and cross off bad options

Forward checking: Cross off values that violate a constraint when added to the existing assignment



Ordering

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Ordering: Minimum Remaining Values

Variable Ordering: Minimum remaining values (MRV):

Choose the variable with the fewest legal values left in its domain

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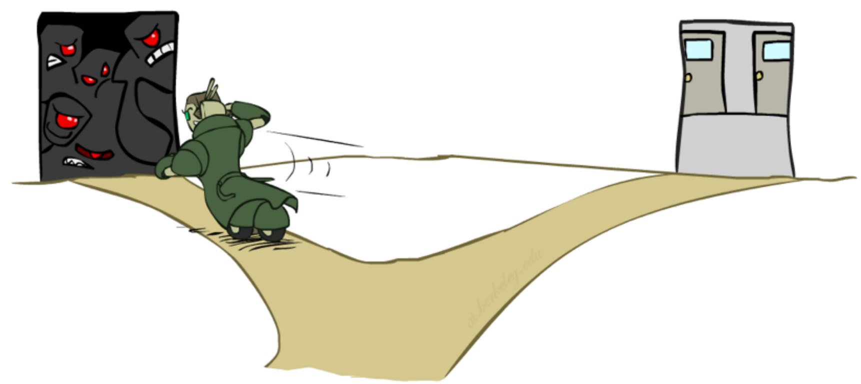
Ordering: Minimum Remaining Values

Why min rather than max?

Also called "most constrained variable"

"Fail-fast" ordering



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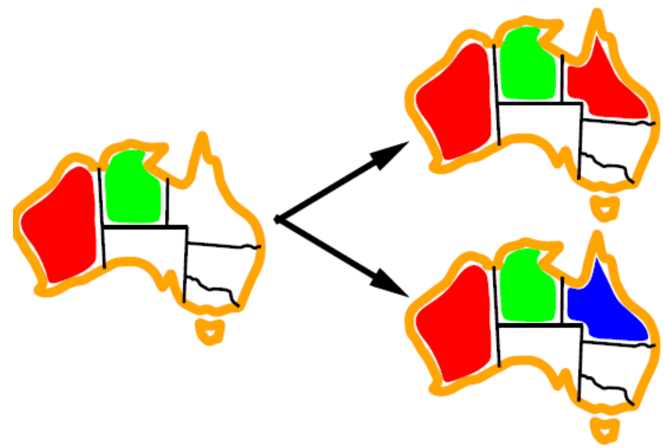
Ordering: Least Constraining Value

Value Ordering: Least Constraining Value



- Given a choice of variable, choose the least constraining value
- For example, the one that rules out the fewest values in the remaining variables
- Note that it may take some computation to determine this (e.g., rerunning filtering)

Ordering: Least Constraining Value

Why least rather than most?

- Combining these ordering ideas makes problems like 1000 queens feasible

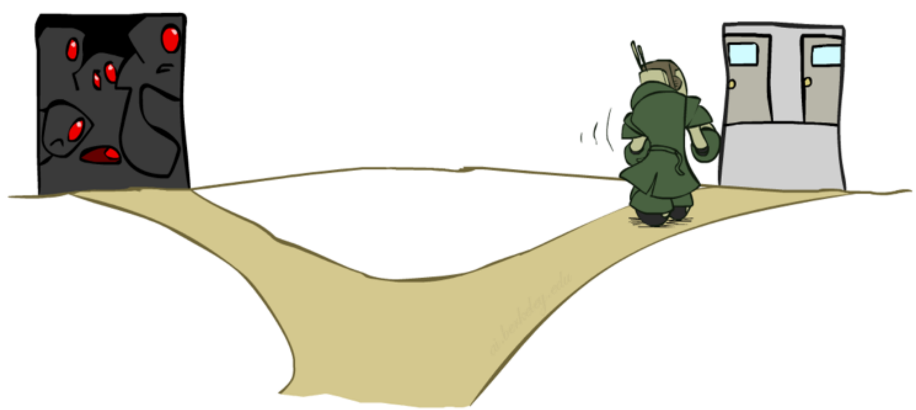


Image credit: Dan Klein and Pieter Abbeel, UC Berkeley CS 188

Structure

- These solutions we’ve seen before:
    - Check the consistency of a single edge
    - Check the edge consistency of an entire CSP
        - **The AC-3 algorithm!**