### CSE 40171: Artificial Intelligence



### Neural Network Model Search: Hyperparameter Optimization Strategies

### Homework #4 has been released It is due at 11:59PM on 10/18

### Quiz #1 is scheduled for 10/30

Project proposal instructions have been released. Proposals are Due 11/4.

(Let me know if you need a group)

Q1: What properties would an approach that is better than random search have?

## Q2: What are some possible alternatives to random search?

### Generic Sequential Model-based Optimization

 $\begin{aligned} & \text{SMBO}(f, M_0, T, S) \\ & 1 & \mathcal{H} \leftarrow \emptyset, \\ & 2 & \text{For } t \leftarrow 1 \text{ to } T, \\ & 3 & x^* \leftarrow \operatorname{argmin}_x S(x, M_{t-1}), \\ & 4 & \text{Evaluate } f(x^*), \qquad \triangleright \text{Expensive step} \\ & 5 & \mathcal{H} \leftarrow \mathcal{H} \cup (x^*, f(x^*)), \\ & 6 & \text{Fit a new model } M_t \text{ to } \mathcal{H}. \\ & 7 & \text{return } \mathcal{H} \end{aligned}$ 

### Tree-Structured Parzen Estimator (TPE)

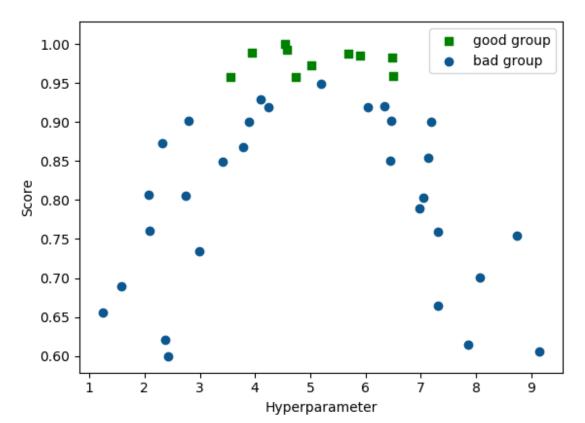
There is more than one way to search via hyperopt:

```
# define an objective function
def objective(args):
    case, val = args
   if case == 'case 1':
       return val
    else:
        return val ** 2
# define a search space
from hyperopt import hp
space = hp.choice('a',
        ('case 1', 1 + hp.lognormal('c1', 0, 1)),
        ('case 2', hp.uniform('c2', -10, 10))
    ])
# minimize the objective over the space
from hyperopt import fmin, tpe, space eval
best = fmin(objective, space, algo=tpe.suggest, max evals=100)
print(best)
# -> { 'a': 1, 'c2': 0.01420615366247227 }
print(space eval(space, best))
# -> ('case 2', 0.01420615366247227}
```

### Step 1: Sample reference sets

Bergstra et al. NIPS 2011

Assumption: "good" and "bad" hyperparameter sets can be modeled by different distributions

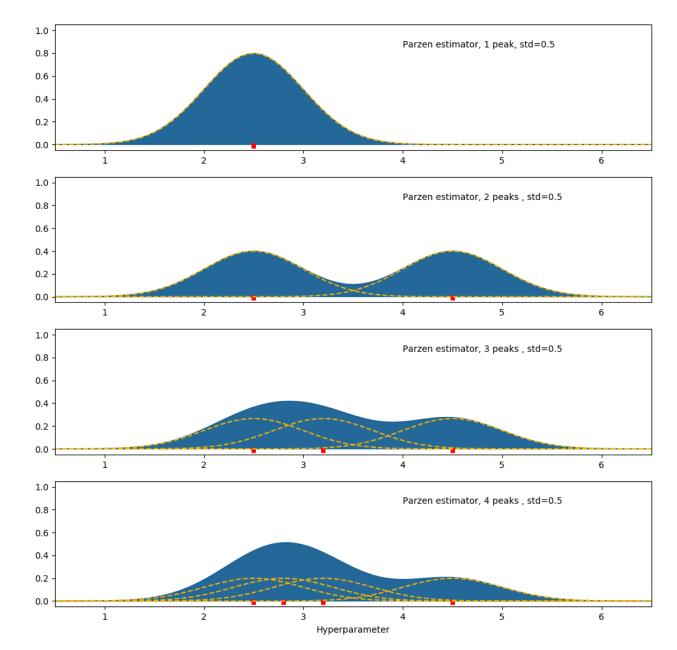


Ex. 25% of parameter sets go to the "good" group after random search

Image Credit: http://dkopczyk.quantee.co.uk/hyperparameter-optimization/

### Step 2: Kernel Density Estimation

- Each sample defines a Gaussian distribution
- Mean equal to a hyperparameter value; specified standard deviation
- Distributions are stacked together and normalized to get a valid prob. distribution



## Step 3: Find candidate with best expected improvement

Sampling Problem: find a hyperparameter combination that more likely belongs to a "good" group and less likely to "bad" group.

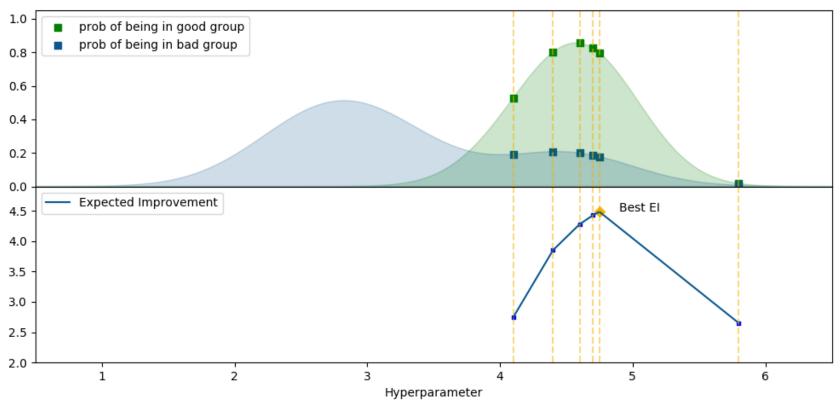


Image Credit: http://dkopczyk.quantee.co.uk/hyperparameter-optimization/

### How effective is TPE?

### Bergstra et al. NIPS 2011

	convex	MRBI
TPE	$\textbf{14.13} \pm 0.30 \ \%$	<b>44.55</b> $\pm 0.44\%$
GP	$16.70\pm0.32\%$	$47.08\pm0.44\%$
Manual	$18.63\pm0.34\%$	$47.39\pm0.44\%$
Random	$18.97 \pm 0.34~\%$	$50.52 \pm 0.44\%$

Algorithms were allowed up to 200 trials. The manual searches used 82 trials for convex and 27 trials MRBI.

### **Boston Housing Price Problem**

(http://dkopczyk.quantee.co.uk/hyperparameter-optimization/)

Random	TIME (minutes)	BEST CV SCORE (%)	TEST SCORE (%)
	3.8	86.10	89.58
TPE	TIME (minutes)	BEST CV SCORE (%)	TEST SCORE (%)
	4.5	86.37	90.42

## Pros of TPE

- + Conceptually Simple
- + Implementations in multiple Python packages
  - hyperopt, optunity
- + At least as effective as random search in some settings

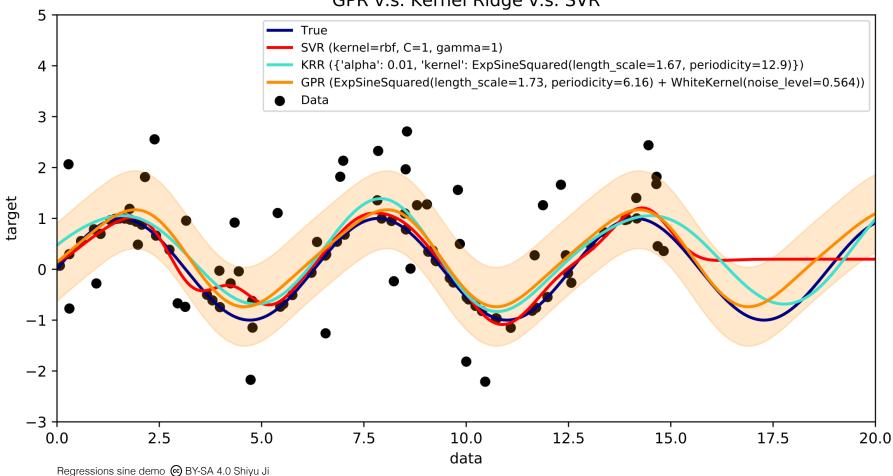
## Cons of TPE

- Limited by the structure learned from data
- It is possible for TPE to be arbitrarily bad with a bad choice of P(y | x)
- Possible to be slower than random sampling at finding a global optimum with an apparently good P(y | x)
- Realistically, will only find a local optimum

### Gaussian Process (GP)

Bergstra et al. NIPS 2011

Long recognized as a good method for modeling loss functions in model-based optimization literature



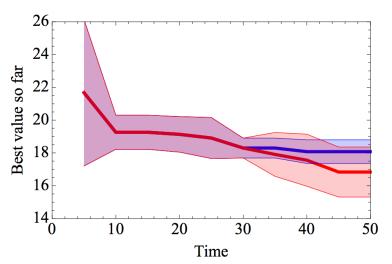
GPR v.s. Kernel Ridge v.s. SVR

### How effective is GP?

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**Boston Housing Price Problem** 

Red = GP, Blue = Random. Shaded areas = one-sigma error bars

### Pros and cons of GP

- + Priors over functions that are *closed under sampling*
- + Provide an assessment of prediction uncertainty incorporating the effect of data scarcity
- + At least as effective as random search in some settings
- Has its own hyperparameters, which must be tuned
- Limited by the structure learned from data
- Realistically, will only find a local optimum
- Empirically worse than TPE

### More on Bayesian Optimization

Let's revisit the idea of Gaussian Process...

Snoek et al. NIPS 2012

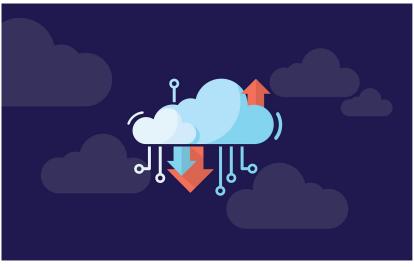
- A learning algorithm's generalization performance is modeled as a sample from a GP
- Type of kernel and the treatment of its hyperparameters, can play a crucial role in obtaining a good optimizer

### But need to factor in variable cost

# Thinking about this problem from a systems perspective

Machine learning problems are different from other black-box optimization problems

• each function evaluation can require a variable amount of time



Cloud computing 🕞 BY 2.0 Jane Boyko

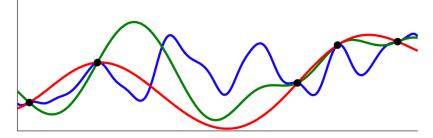
Machine learning experiments are often run in parallel, on multiple cores or machines.

Problem: in both cases the standard sequential approach of GP optimization can be suboptimal

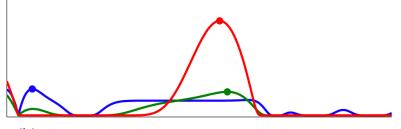
### Integrated acquisition function

$$\hat{a}(\mathbf{x}; \{\mathbf{x}_n, y_n\}) = \int a(\mathbf{x}; \{\mathbf{x}_n, y_n\}, \theta) p(\theta \mid \{\mathbf{x}_n, y_n\}_{n=1}^N) d\theta,$$

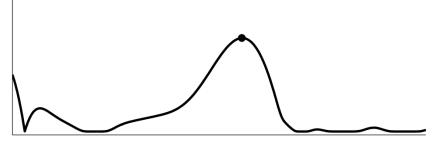
Depends on the parameters and all of the observations



(a) Posterior samples under varying hyperparameters



(b) Expected improvement under varying hyperparameters



(c) Integrated expected improvement

# Monte Carlo acquisition for parallelizing Bayesian optimization

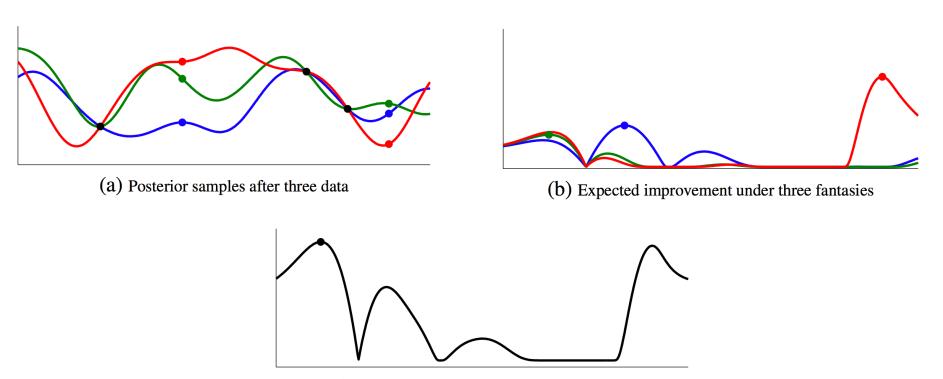
Compute Monte Carlo estimates of the acquisition function under different possible results from pending function evaluations.

Scenario: *N* evaluations have completed, yielding data  $\{\mathbf{x}_n, y_n\}_{n=1}^N$ , and in which *J* evaluations are pending at locations  $\{\mathbf{x}_j\}_{j=1}^J$ 

Choose a new point based on the expected acquisition function under all possible outcomes of these pending evaluations:

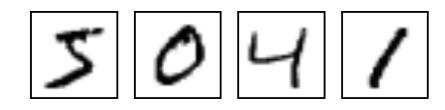
$$\hat{a}(\mathbf{x}; \{\mathbf{x}_{n}, y_{n}\}, \theta, \{\mathbf{x}_{j}\}) = \int_{\mathbb{R}^{J}} a(\mathbf{x}; \{\mathbf{x}_{n}, y_{n}\}, \theta, \{\mathbf{x}_{j}, y_{j}\}) p(\{y_{j}\}_{j=1}^{J} | \{\mathbf{x}_{j}\}_{j=1}^{J}, \{\mathbf{x}_{n}, y_{n}\}_{n=1}^{N}) dy_{1} \cdots dy_{J}.$$

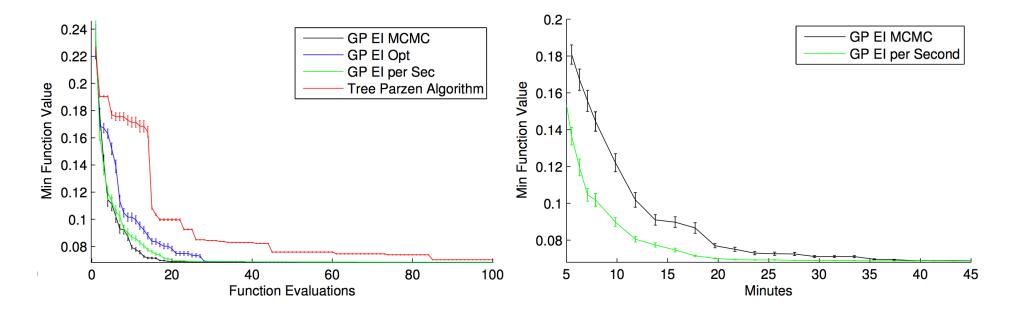
## Monte Carlo acquisition for parallelizing Bayesian optimization



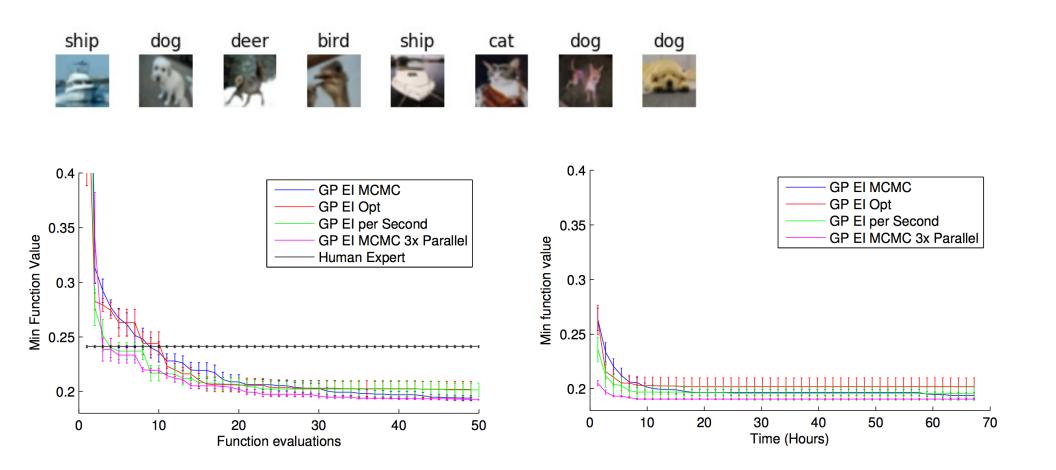
(c) Expected improvement across fantasies

### Logistic regression on MNIST





### CNN validation error on CFAR-10



## Spearmint

#### $\leftarrow \rightarrow$ C ( GitHub, Inc. [US] | https://github.com/HIPS/Spearmint

Spearmint Bayesian optimization codebase

P 97 commits	ဖို <b>3</b> branches	$\bigcirc$ 0 releases	<b>L</b> 11 contributors	ৰ্ষ্যু View license
Branch: master ▼ New pull r	equest		[	Find File Clone or download -
mgelbart Merge pull request	#121 from jjerphan/patch-1			Latest commit 990d27d on Apr 2
examples	removed non-default grid	d size from config file of noisy	function, add	5 years ago
spearmint	solved issue #32: Simple	Case of 1 Optimization Variab	ble	3 years ago
.gitignore	initial commit			5 years ago
	Update CONTRIBUTING.	rst		5 years ago
LICENSE.md	Update LICENSE.md			5 years ago
README.md	Fix README.md format			3 months ago
Contributors.md	Update contributors.md			4 years ago
🖹 setup.py	Fixed a couple of issues			5 years ago
E README.md				

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#### Spearmint

Spearmint is a software package to perform Bayesian optimization. The Software is designed to automatically run experiments (thus the code name spearmint) in a manner that iteratively adjusts a number of parameters so as to minimize some objective in as few runs as possible.

## Optunity

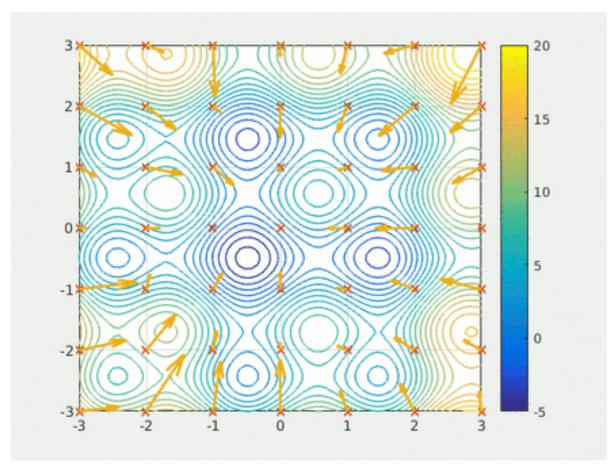


https://github.com/claesenm/optunity.git

pip install optunity

### Particle Swarm Optimization

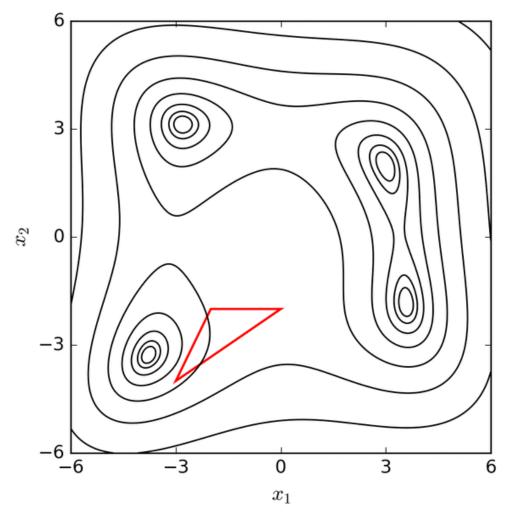
Position of a particle represents a set of hyperparameters Movement is influenced by the goodness of the objective function value



Particle Swarm Arrows Animation 💿 BY-SA 4.0 Ephramac

### Nelder-Mead Simplex

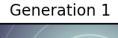
- Nonlinear optimization method based on the concept of a *simplex*
- Good local search method, but will get stuck in bad regions when a poor starting point is specified

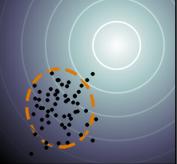


Nelder-Mead Himmelblau @BY 4.0 Nicoguaro

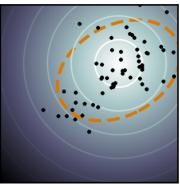
### Covariance Matrix Adaptation Evolutionary Strategy

- Evolutionary strategy for continuous function optimization
- Dynamically adapt search resolution per hyperparameter, allowing for efficient searches at different scales

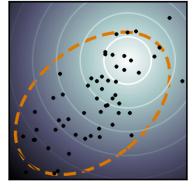




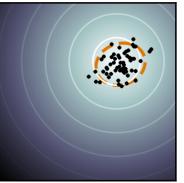
Generation 4



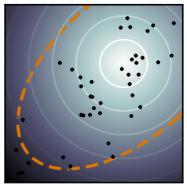
Generation 2



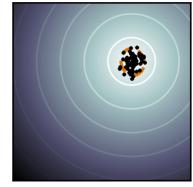
Generation 5



Generation 3

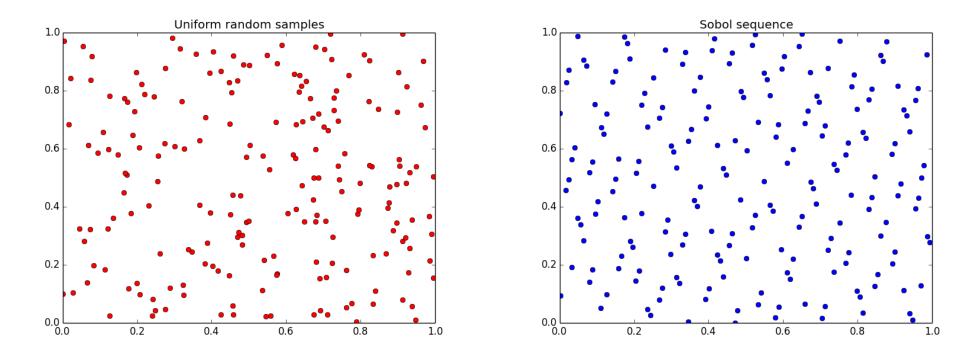


Generation 6



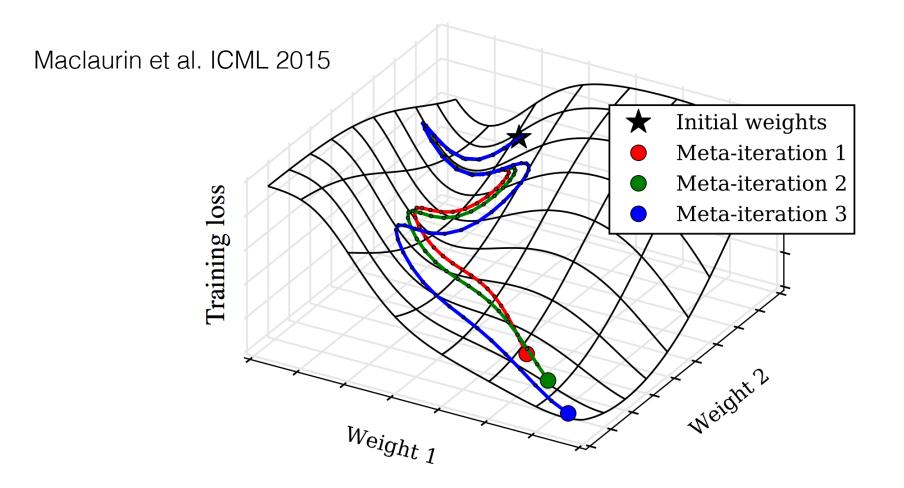
## Sobol Sequences

Sobol sequences are designed to cover the unit hypercube with lower discrepancy than completely random sampling



Plots generated via optunity: https://optunity.readthedocs.io

### Gradient-based Optimization



- 1. Entire training run with SGD to optimize weights
- 2. Compute gradients of validation loss with respect to hyperparameters via backprop.
- 3. Update hyperparameters in the direction of this hypergradient

### Opens up a "garden of delights"...

- Efficient optimization of thousands of parameters
- Finer-grained hyperparameter optimization, *e.g.*, per layer optimization in a neural network
- Flexibility over:
  - Model classes
  - Regularization
  - Training Methods

### Optimization over training error (MNIST)

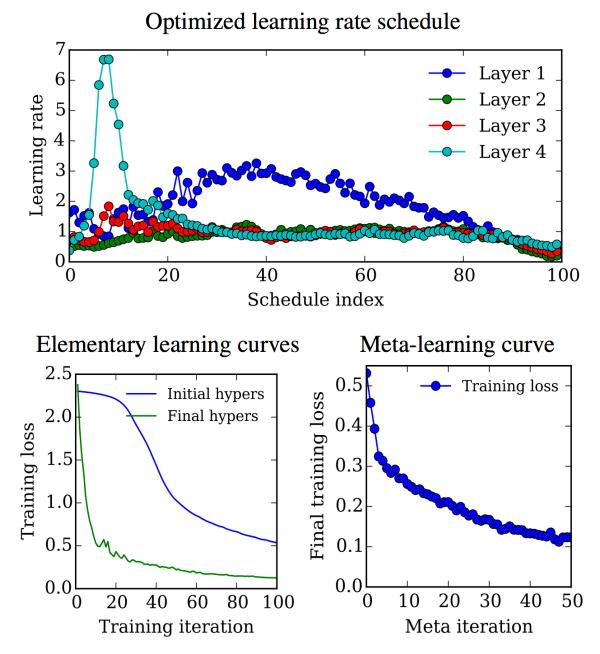


Image Credit: Maclaurin et al. ICML 2015

### Limitations

- Learning long-term dependencies with gradient descent is difficult
  - Large learning rates induce chaotic behavior in the learning dynamics
- Overfitting
- Discrete parameters still need to be optimized by hand

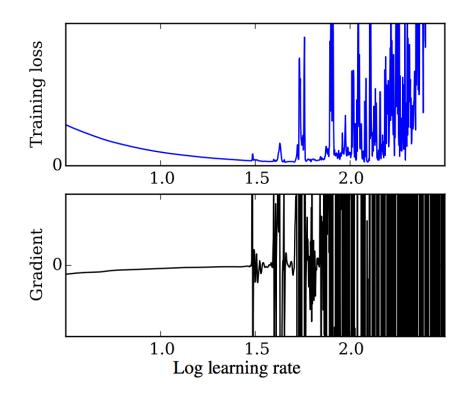


Image Credit: Maclaurin et al. ICML 2015