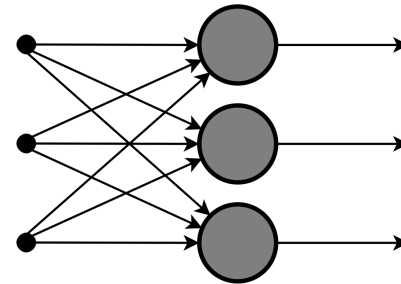
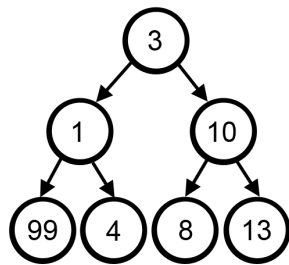


# CSE 40171: Artificial Intelligence



Artificial Neural Networks with Anatomical Fidelity:  
Recurrence in Artificial and Biological Networks

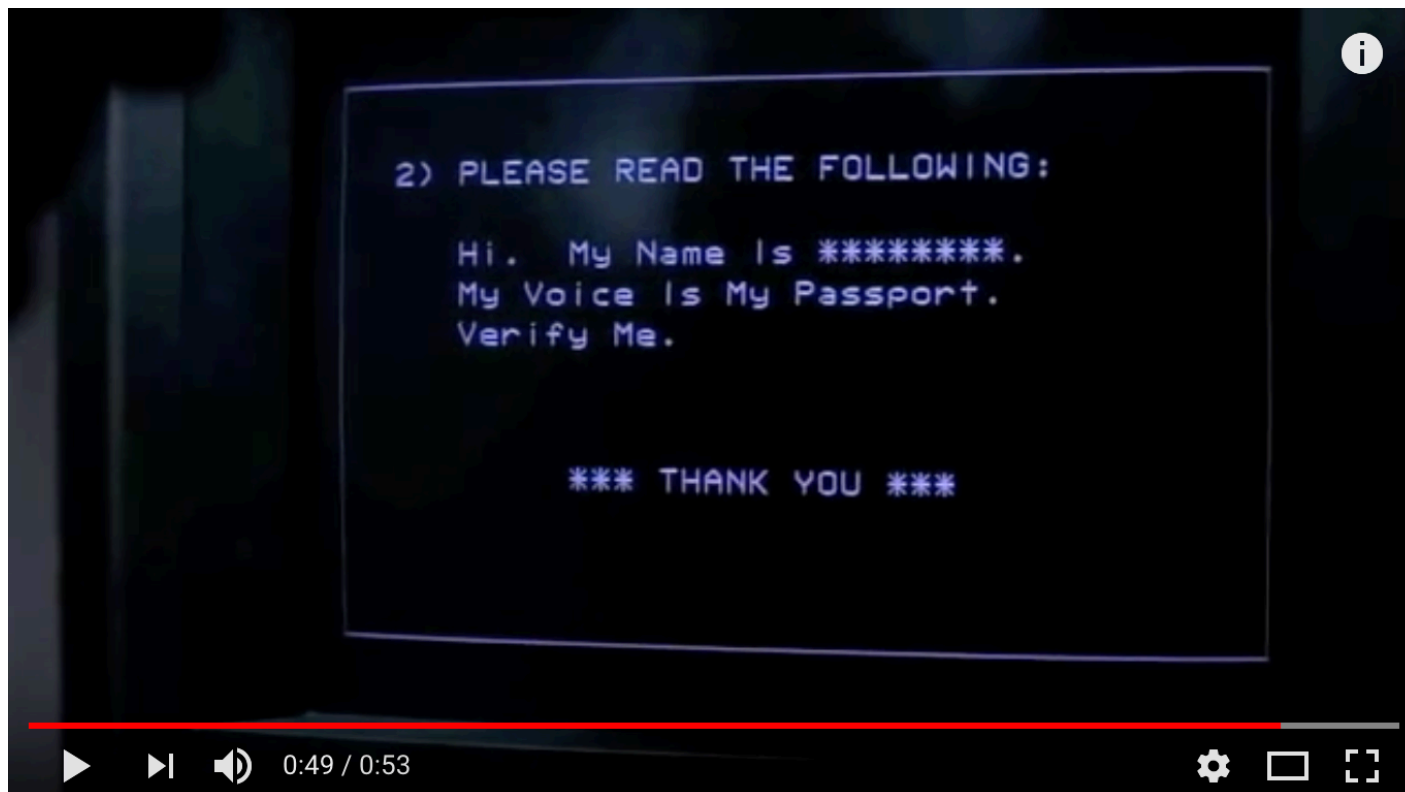
Homework #6 has been released  
It is due at 11:59PM on 11/22

Project Updates are Due on 11/25 at  
11:59PM

(See Course Website for Instructions)

Let's turn our attention back to artificial neural networks for a moment...

# How do we parse audio data?

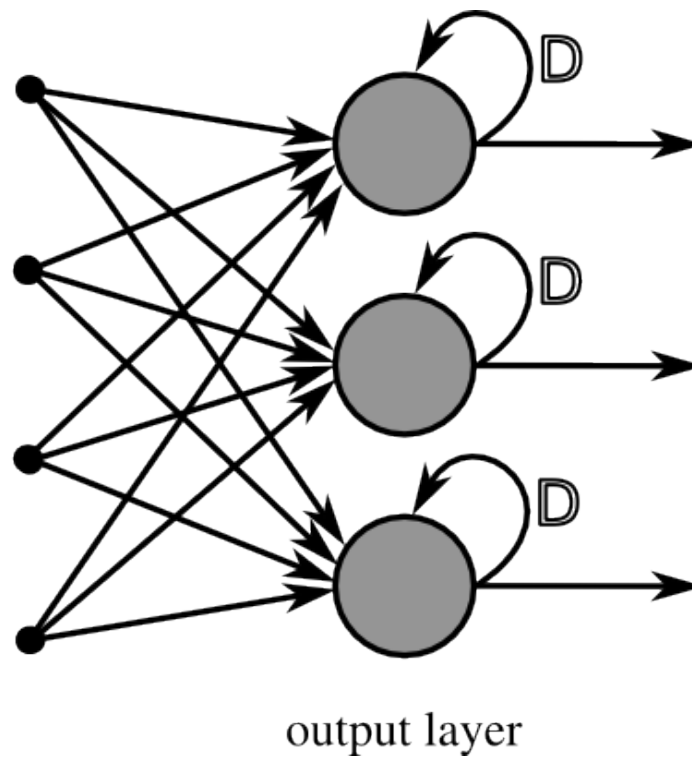


<https://www.youtube.com/watch?v=-zVgWpVXb64>

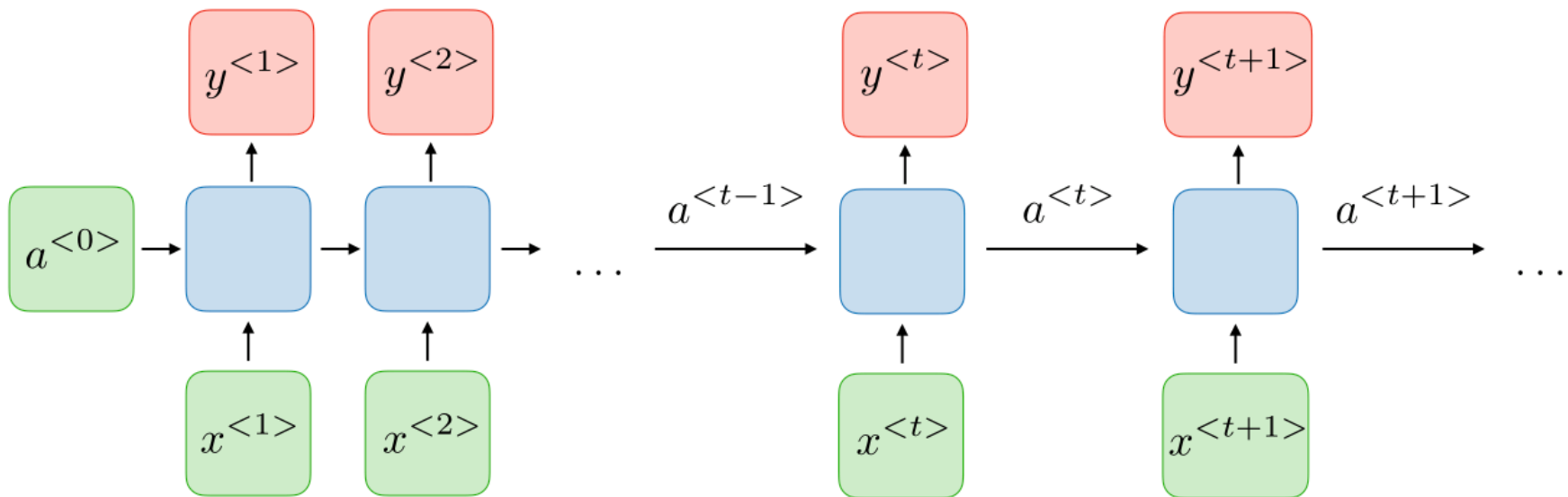
# How about text?

Arms, and the man I sing, who, forc'd by fate,  
And haughty Juno's unrelenting hate,  
Expell'd and exil'd, left the Trojan shore.  
Long labors, both by sea and land, he bore,  
And in the doubtful war, before he won  
The Latian realm, and built the destin'd town;  
His banish'd gods restor'd to rites divine,  
And settled sure succession in his line,  
From whence the race of Alban fathers come,  
And the long glories of majestic Rome.  
O Muse! the causes and the crimes relate;  
What goddess was provok'd, and whence her hate;  
For what offense the Queen of Heav'n began  
To persecute so brave, so just a man;  
Involv'd his anxious life in endless cares,  
Expos'd to wants, and hurried into wars!

# Recurrent Networks



# Architecture of a Traditional RNN



# At each timestep $t$ :

Activation  $a^{<t>}$ :

$$a^{<t>} = g_1(W_{aa}a^{<t-1>} + W_{ax}x^{<t>} + b_a)$$

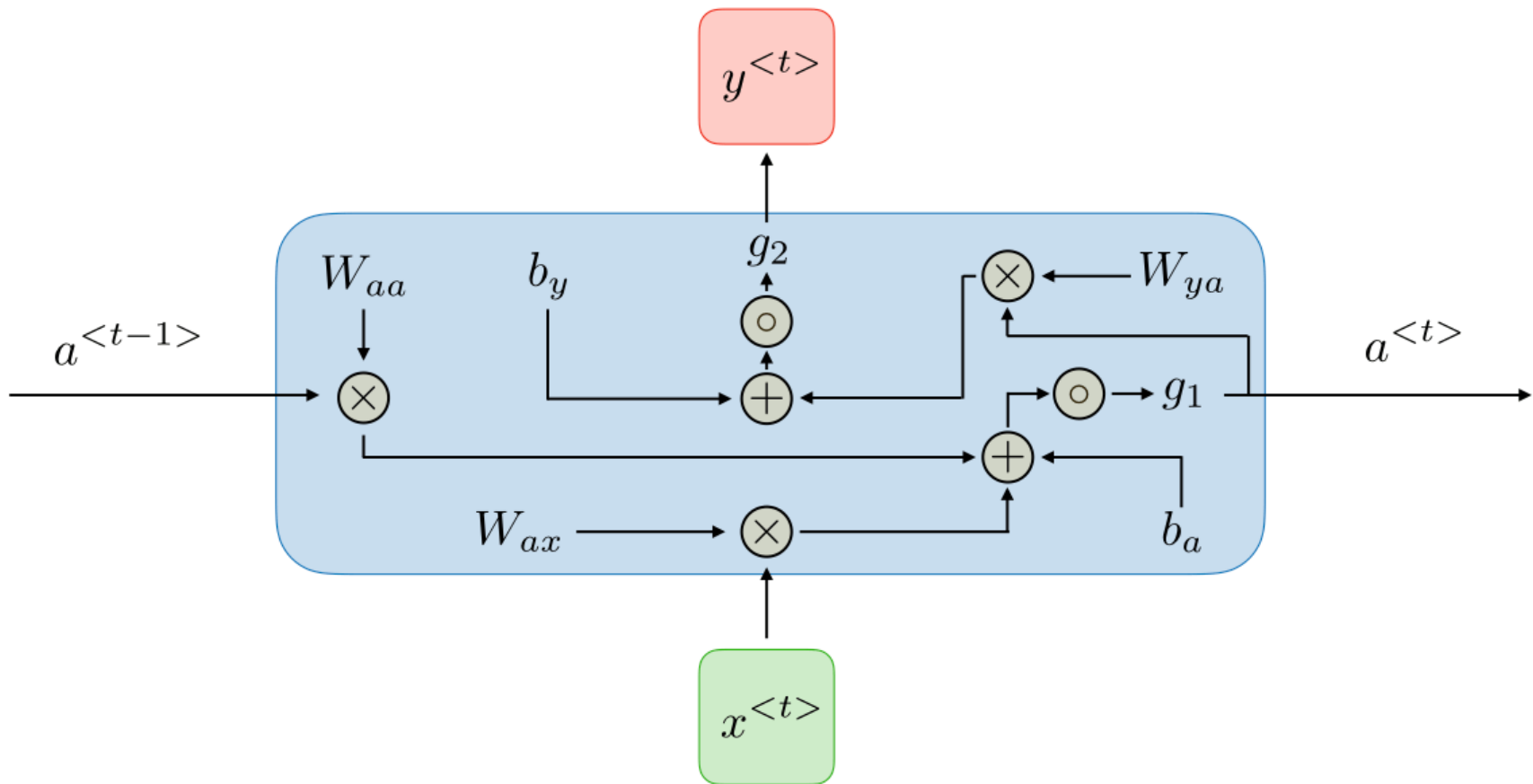
**activation function**

**coefficient**

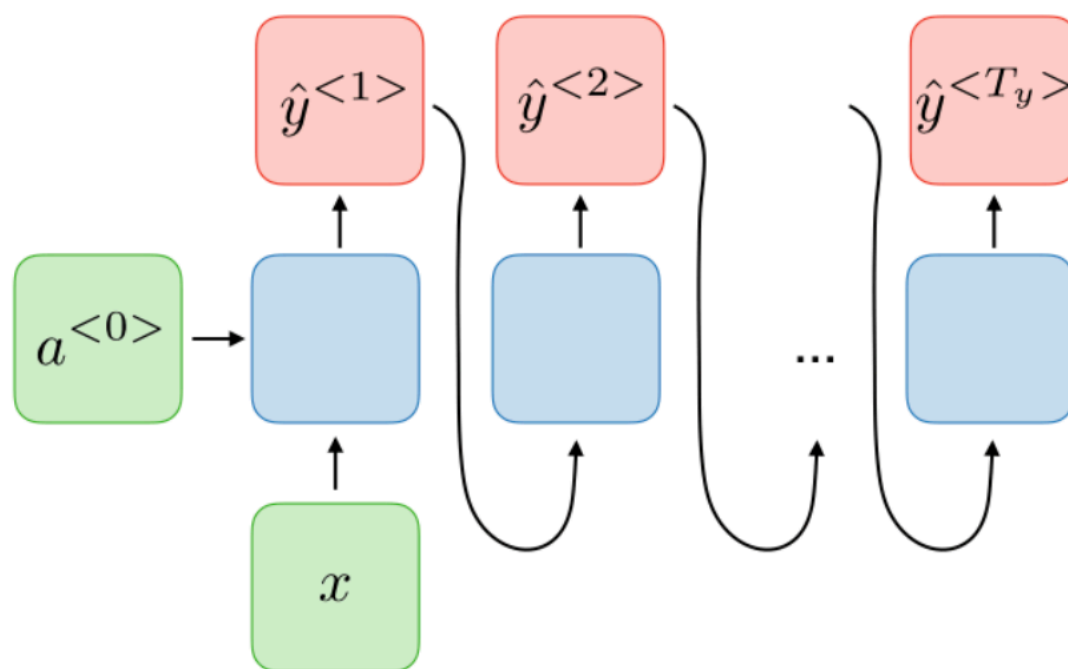
Output  $y^{<t>}$ :

$$y^{<t>} = g_2(W_{ya}a^{<t>} + b_y)$$

At each timestep  $t$ :

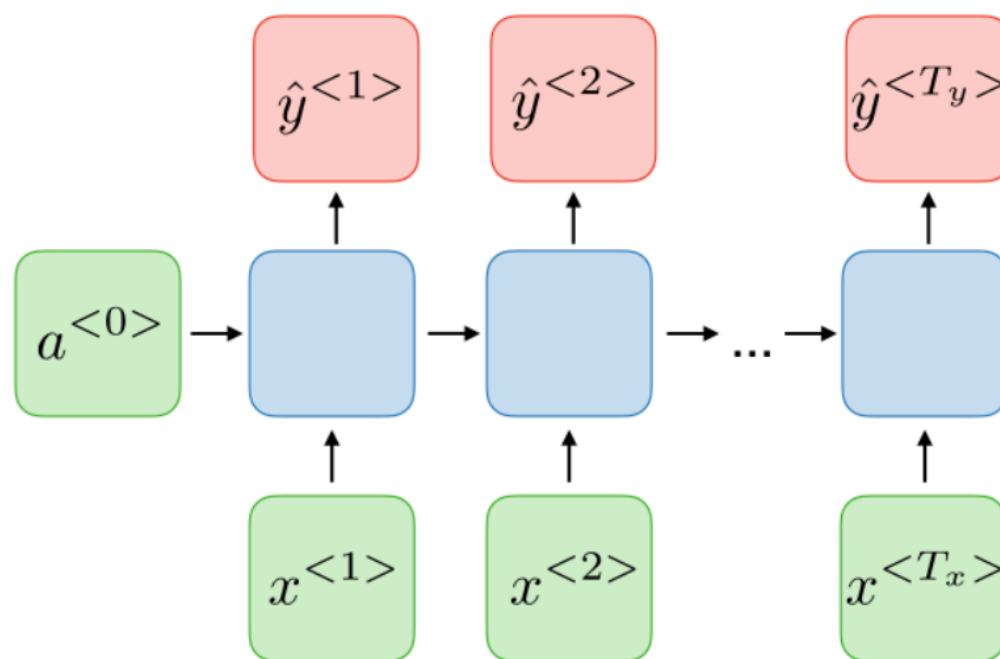


# Example Application: Music Generation



One-to-Many,  $T_x = 1, T_y > 1$

# Example Application: Named Entity Recognition



Many-to-Many,  $T_x = T_y$

# Training a Recurrent Net

Loss Function:

$$\mathcal{L}(\hat{y}, y) = \sum_{t=1}^{T_y} \mathcal{L}(\hat{y}^{<t>}, y^{<t>})$$

Prediction

Ground-truth

Time Step

Backprop:

$$\frac{\partial \mathcal{L}^{(T)}}{\partial W} = \sum_{t=1}^T \frac{\partial \mathcal{L}^{(T)}}{\partial W} \Big|_{(t)}$$

Weight Matrix

# Advantages of RNNs

- Ability to process time series data of any length
- Model size does not increase with input size
- Takes into account history when processing
- Weights are shared across time

# Disadvantages of RNNs

- Computation is slow
- Difficulty accessing information from many timesteps in the past
- Current state cannot consider any input from the future

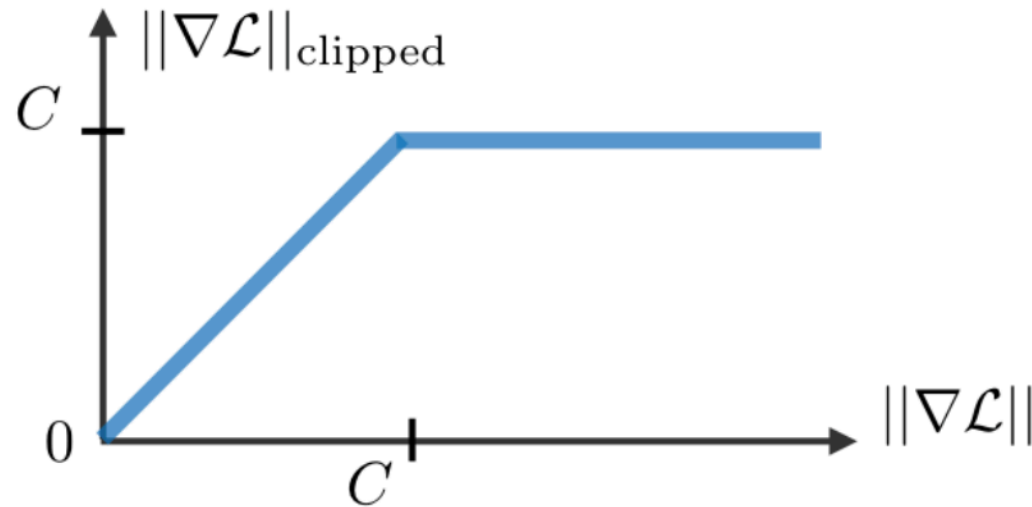
# Vanishing / Exploding Gradient

For many years, RNNs were interesting theoretically, but not practical to train

Reason: it is difficult to capture long term dependencies because of a multiplicative gradient that can be exponentially increasing / decreasing with respect to the number of layers in the network

# RNN Strategy 1: Gradient Clipping

Cap the maximum value of the gradient:



# RNN Strategy 2: Types of Gates

Specific gates that have a well-defined purpose can address the vanishing gradient problem.

General Form:

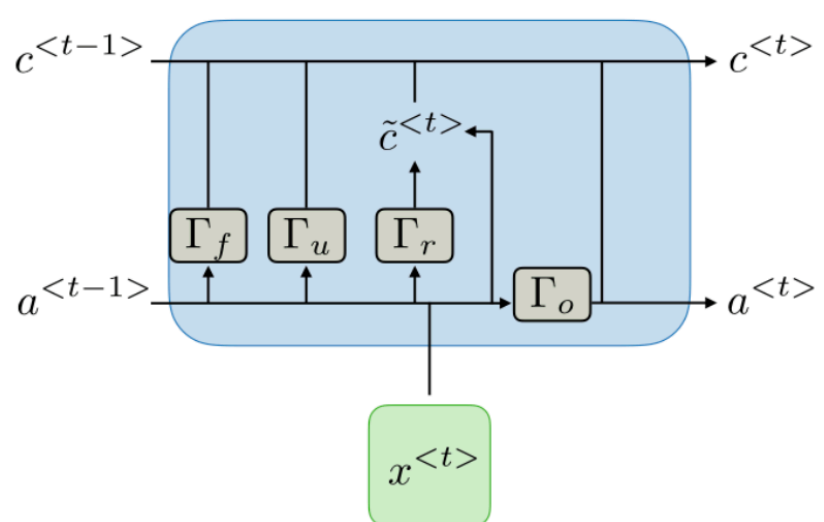
$$\Gamma = \sigma(Wx^{<t>} + Ua^{<t-1>} + b)$$

Sigmoid Function

Gate-Specific Coefficients

Type of gate	Role
Update gate $\Gamma_u$	How much past should matter now?
Relevance gate $\Gamma_r$	Drop previous information?
Forget gate $\Gamma_f$	Erase a cell or not?
Output gate $\Gamma_o$	How much to reveal of a cell?

# Standard Arch. That Works: LSTM

Characterization	Long Short-Term Memory (LSTM)
$\tilde{c}^{<t>}$	$\tanh(W_c[\Gamma_r \star a^{<t-1>}, x^{<t>}] + b_c)$
$c^{<t>}$	$\Gamma_u \star \tilde{c}^{<t>} + \Gamma_f \star c^{<t-1>}$
$a^{<t>}$	$\Gamma_o \star c^{<t>}$
Dependencies	 <p>The diagram illustrates the internal structure of an LSTM cell. It shows the flow of information from the previous time step to the current one. The inputs are the previous hidden state <math>c^{&lt;t-1&gt;}</math> and the previous cell state <math>a^{&lt;t-1&gt;}</math>. The current input <math>x^{&lt;t&gt;}</math> is fed into the cell. The cell contains four gates: <math>\Gamma_f</math> (forget gate), <math>\Gamma_u</math> (update gate), <math>\Gamma_r</math> (reset gate), and <math>\Gamma_o</math> (output gate). The reset gate <math>\Gamma_r</math> and the current input <math>x^{&lt;t&gt;}</math> are combined to produce the candidate cell state <math>\tilde{c}^{&lt;t&gt;}</math>. The forget gate <math>\Gamma_f</math> and the candidate cell state <math>\tilde{c}^{&lt;t&gt;}</math> are combined to produce the new cell state <math>c^{&lt;t&gt;}</math>. The update gate <math>\Gamma_u</math> and the new cell state <math>c^{&lt;t&gt;}</math> are combined to produce the new cell state <math>c^{&lt;t&gt;}</math>. The output gate <math>\Gamma_o</math> and the new cell state <math>c^{&lt;t&gt;}</math> are combined to produce the new cell state <math>a^{&lt;t&gt;}</math>.</p>

# Recurrent Models of Brain Function

# Two models of recurrence

Linear recurrent model:

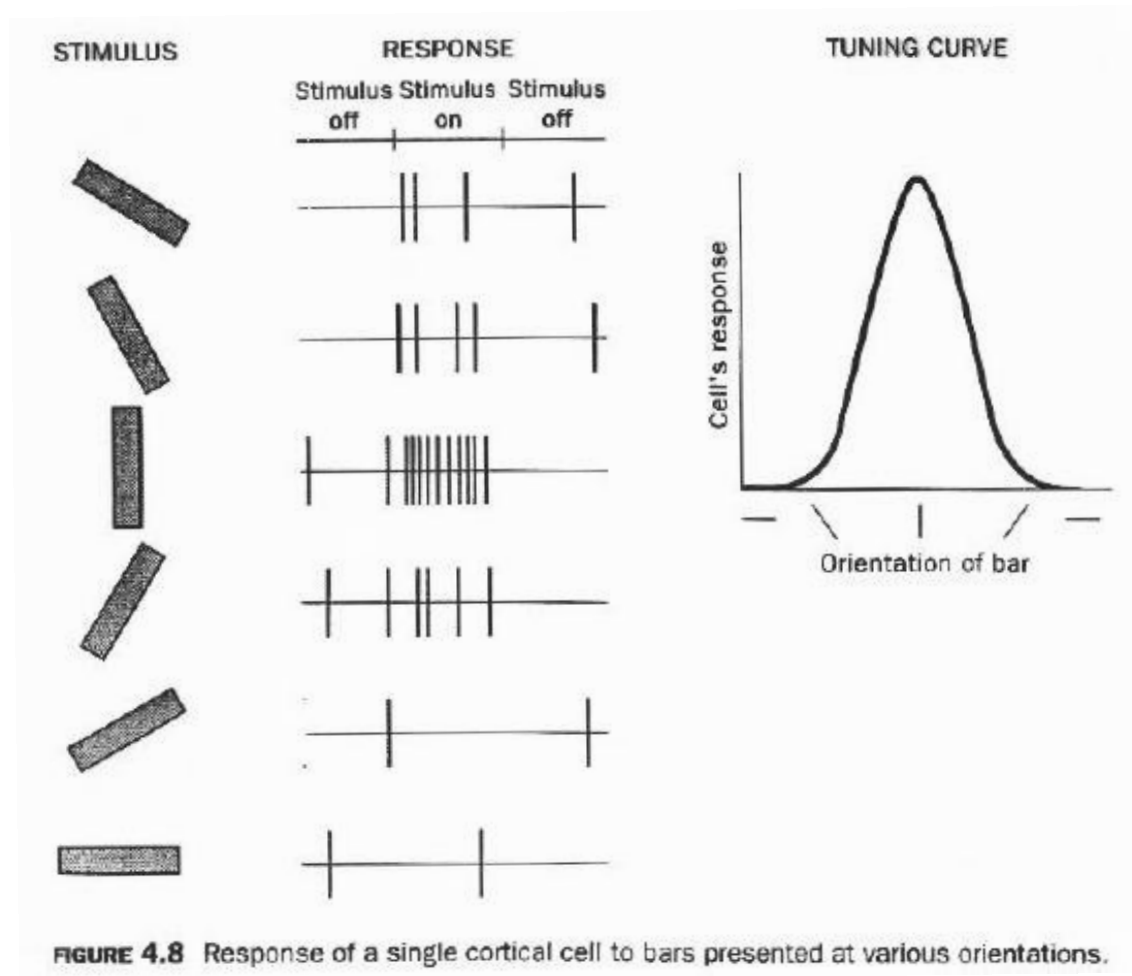
$$\tau_r \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{F}(\mathbf{h} + \mathbf{M} \cdot \mathbf{v})$$

Non-linear recurrent model:

$$\mathbf{F}(\mathbf{h} + \mathbf{M} \cdot \mathbf{r}) = [\mathbf{h} + \mathbf{M} \cdot \mathbf{r} - \gamma]_+$$

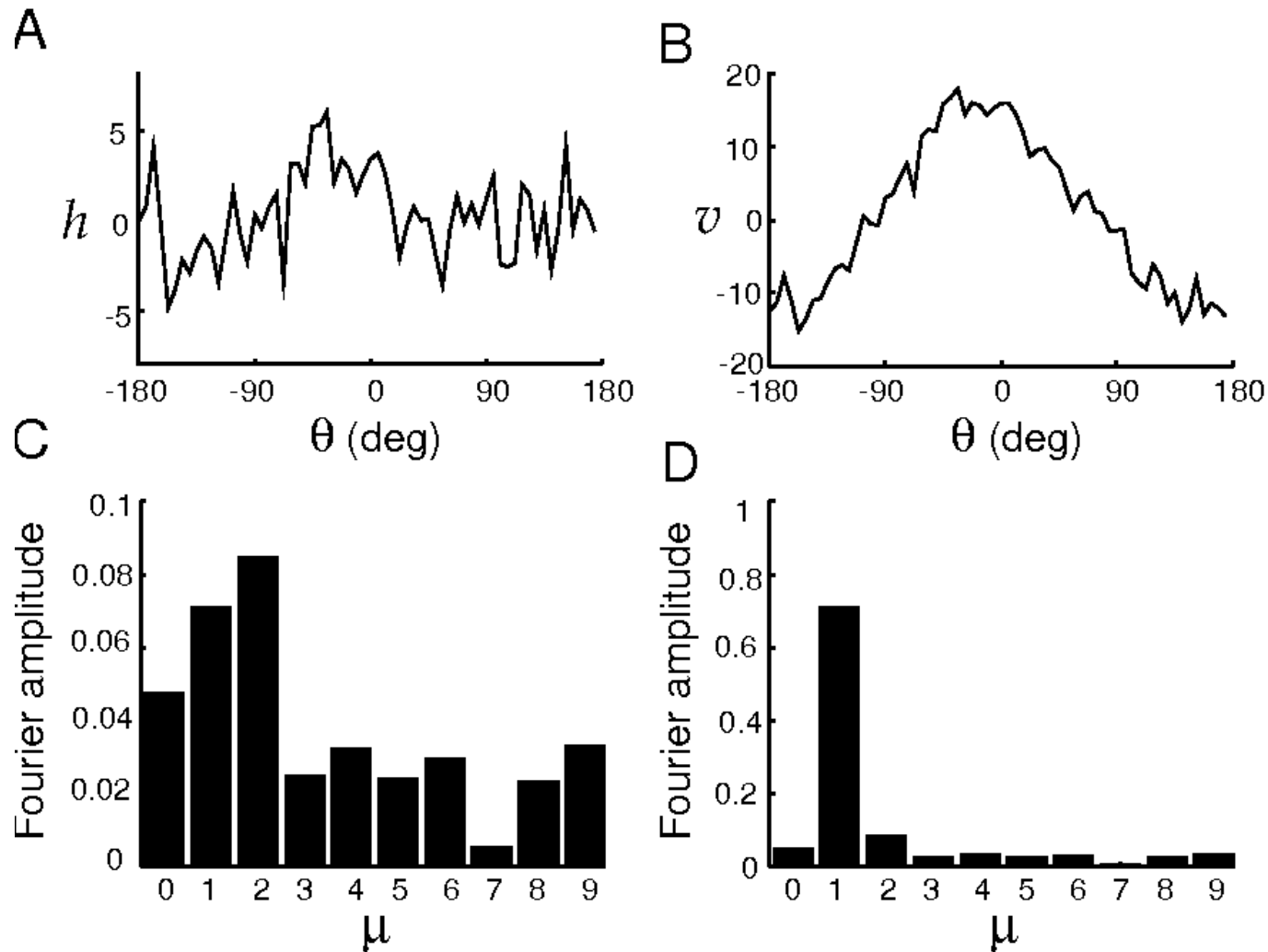
↑  
Adds rectification

# Tuning Curves

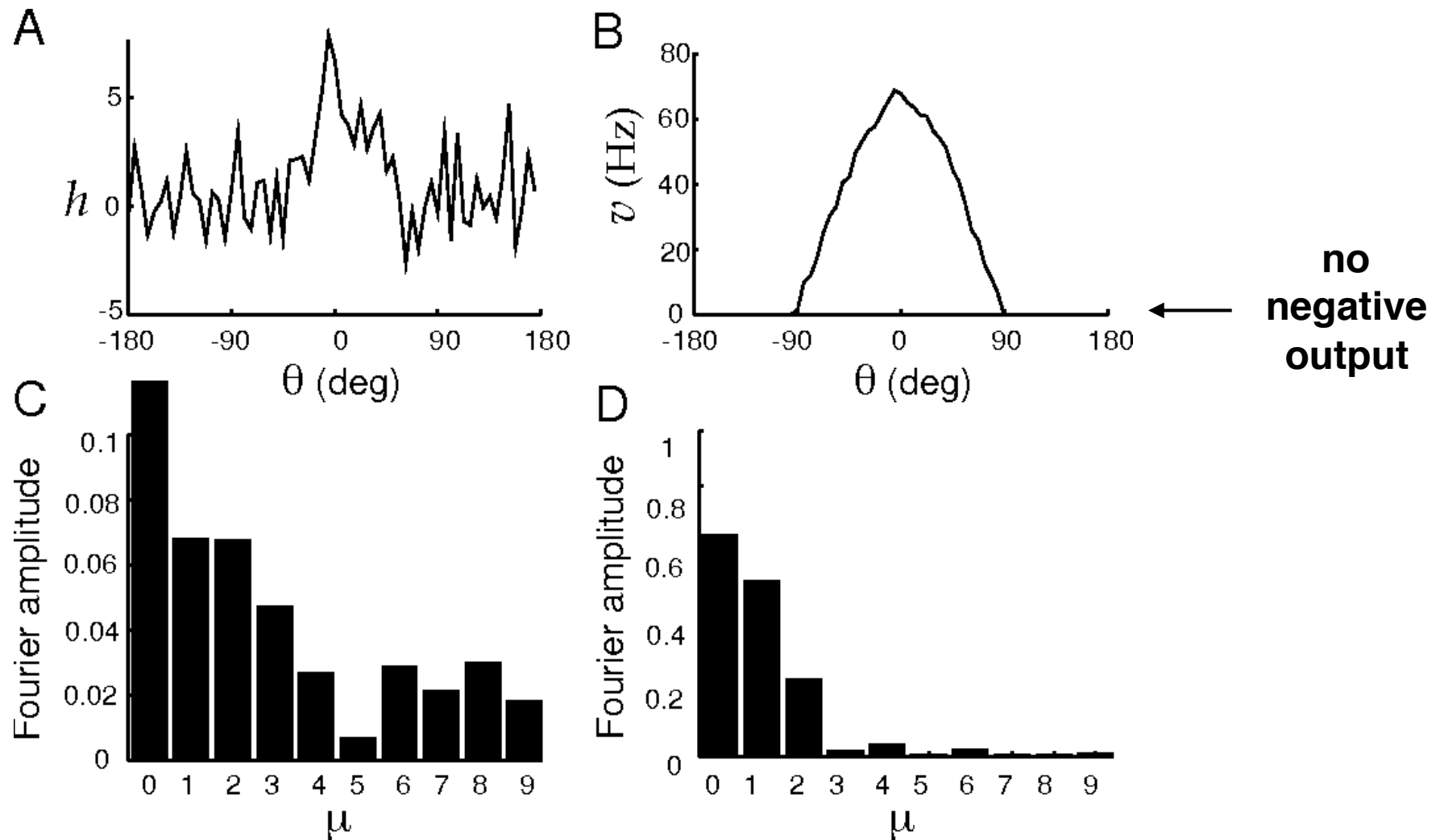


Hubel and Wiesel 1968

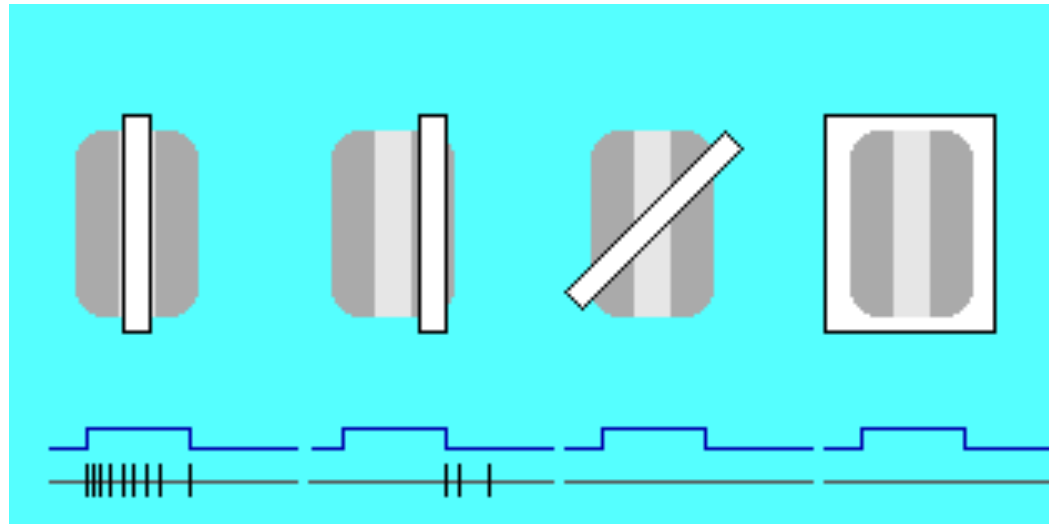
# Linear Amplification



# Non-Linear Amplification



# Recurrent Model of Simple Cells in Primary Visual Cortex



# Recurrent Model of Simple Cells in Primary Visual Cortex

Ben-Yishai, Bar-Or and Sompolinsky 2005

firing rate of neuron

orientation

clamped orientation angles

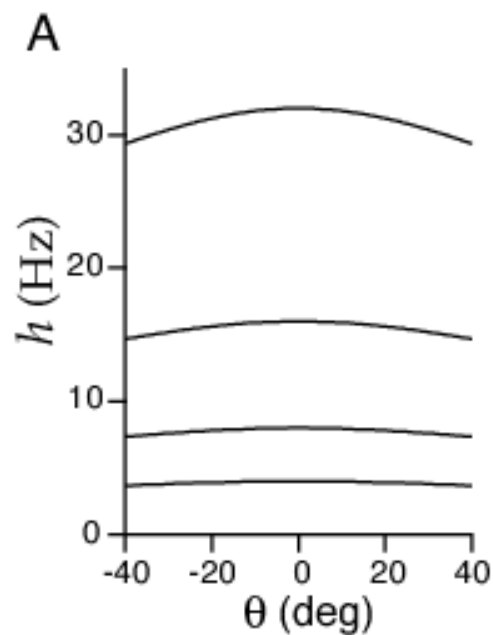
$$\tau_r \frac{dv(\theta)}{dt} = -v(\theta) + \left[ h(\theta) + \int_{-\pi/2}^{\pi/2} \frac{d\theta'}{\pi} (-\lambda_0 + \lambda_1 \cos(2(\theta - \theta'))) v(\theta') \right]_+$$

image contrast

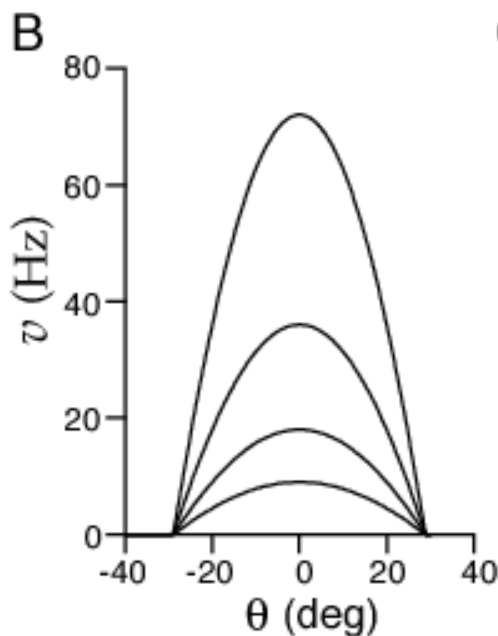
overall amplitude

$$h(\theta) = Ac(1 - \epsilon + \epsilon \cos(2\theta))$$

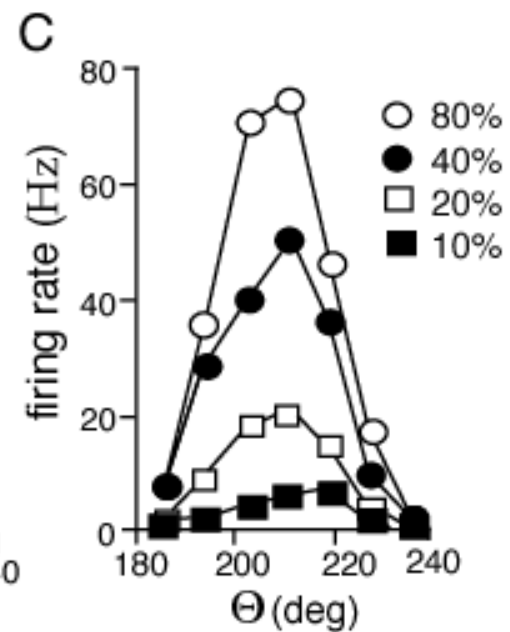
# Recurrent Model of Simple Cells in Primary Visual Cortex



Feed-Forward  
Model

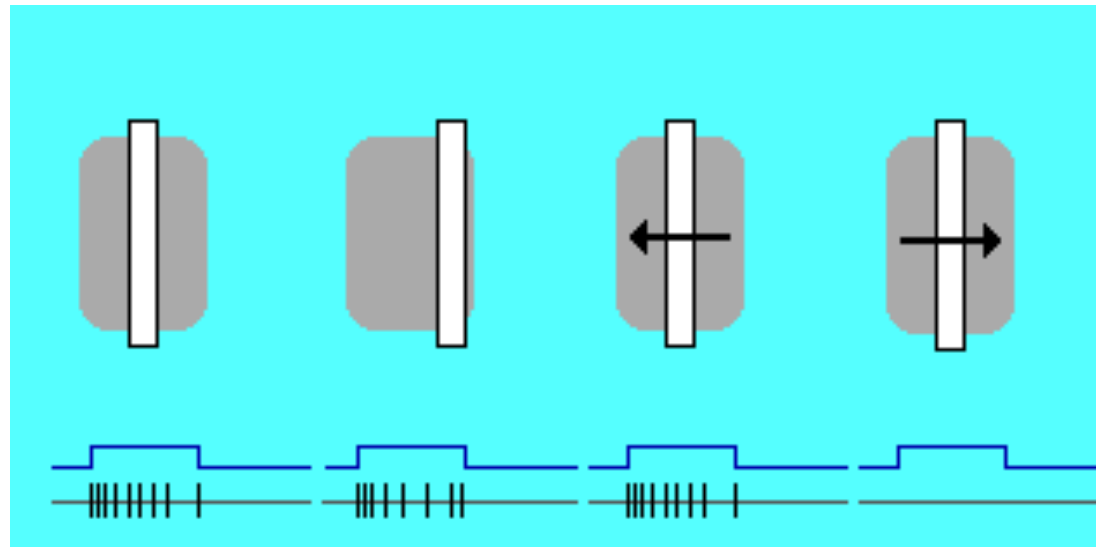


Recurrent  
Model



Experimental  
Data

# A Recurrent Model of Complex Cells in Primary Visual Cortex



# A Recurrent Model of Complex Cells in Primary Visual Cortex

Chance, Nelson and Abbott 1999

Weight Function:

$$M(\phi - \phi') = \lambda_1 / (2\pi \rho_\phi)$$

angle

control recurrence

Firing Rates Determined By:

$$\tau_r \frac{dv(\phi)}{dt} = -v(\phi) + \left[ h(\phi) + \frac{\lambda_1}{2\pi} \int_{-\pi}^{\pi} d\phi' v(\phi') \right]_+$$

feed-forward input

# A Recurrent Model of Complex Cells in Primary Visual Cortex

