CSE 40567 / 60567: Computer Security

Cryptography 3
Homework #2 has been released. It is due Thursday, Feb. 7th at 11:59PM

See Assignments Page on the course website for details
Cryptographic Algorithms
Random Number Generation

“Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin”

- Knuth quoting von Neumann

• Where do random numbers come from?
  ‣ Produced by computers (Pseudo-random)
  ‣ Measured from physical phenomena (Real Random)
Pseudo-random Sequences

1. It looks random. Passes all statistical tests of randomness that we can find*

2. It is unpredictable. Computationally infeasible to predict what the next random bit will be, given complete knowledge of the algorithm, hardware, and all previous bits

\[
s_1, f(s_1) = 01011110\ldots \quad \text{• Seed } s_x \text{ is treated as a secret}
\]
\[
s_2, f(s_2) = 11010111\ldots \quad \text{• A seed will yield the same sequence when reused}
\]
\[
s_3, f(s_3) = 10101110\ldots
\]

Real Random Sequences

3. It cannot be reliably reproduced. If you run the generator twice with exactly the same input, you will get two completely unrelated random sequences

Sources of entropy

- Timing of read/write head movement
- Noise from a reverse bias transistor, e.g., Raspberry Pi HRNG
Sources of random numbers

- `/dev/random` - blocking pseudo-random number generator (hash implementation via SHA)
  - Handy way to check the entropy pool:
    
    ```bash
    $ cat /proc/sys/kernel/random/entropy_avail
    ```

- `/dev/urandom` - unlimited non-blocking pseudorandom number generator

- Python: `import random` (Mersenne Twister)
  
  ```python
  >>> random.seed(1)
  >>> random.random()
  ```
Random number generation in GPG

$ gpg --gen-key

gpg (GnuPG) 1.4.11; Copyright (C) 2010 Free Software Foundation, Inc. This is free software: you are free to change and redistribute it. There is NO WARRANTY, to the extent permitted by law.

We need to generate a lot of random bytes. It is a good idea to perform some other action (type on the keyboard, move the mouse, utilize the disks) during the prime generation; this gives the random number generator a better chance to gain enough entropy.

Not enough random bytes available. Please do some other work to give the OS a chance to collect more entropy! (Need 284 more bytes)
One-way functions

• Input maps to unique output
• Given $x$, it is easy to compute $f(x)$
• Given $f(x)$, it is hard to compute $x$

Analogy: breaking a window

It’s easy to smash a window into a thousand pieces; not easy to put it back together.
Recall hash functions from Data Structures

- *Hash Tables* are useful data structures that allow searching in $O(1)$ time
  - Facilitated by a hash function

Example:

```c
size_t precision = 2;
size_t hash(const char* str)
{
    return (*(size_t*)str) >> precision;
}
```
Requirements for cryptographically strong hash functions

- Must be **collision-free**: hard to generate two inputs with the same hash value
- The output must not be dependent on the input in any discernible way
- A single bit change in the input changes, on average, half of the bits in the hash value
- Given a hash value, it is computationally infeasible to find the input that hashes to that value
md5

- 128-bit hash function

Commonly used as checksum for downloads:

```bash
$ md5sum ubuntu-14.04.3-desktop-amd64.iso
cab6dd5ee6d649ed1b24e807c877c0ae ubuntu-14.04.3-desktop-amd64.iso
```

- Input: 512-bit blocks, divided into 32-bit sub-blocks

- Output: Set of four 32-bit blocks, concatenated

- After input padding, main loop processes input through four rounds
One md5 round

$F = \text{non-linear function}$

$M_i = \text{sub-block of the message}$

$K_i = \text{constant}$
Non-linear functions

Four non-linear functions, a different one for each round:

\[
F (X, Y, Z) = (X \land Y) \lor ((\neg X) \land Z)
\]
\[
G (X, Y, Z) = (X \land Z) \lor (Y \land (\neg Z))
\]
\[
H (X, Y, Z) = X \oplus Y \oplus Z
\]
\[
I (X, Y, Z) = Y \oplus (X \lor ((\neg Z))
\]

If the corresponding bits of \(X\), \(Y\), and \(Z\) are independent and unbiased, then each bit of the result will also be
Collision Attack

• Note that md5’s output space is finite (128 bits), thus collisions *must* exist
  ‣ But it should be infeasible to find them

• Modular differential attack: Wang and Yu, Eurocrypt 2005

---

given target

... fulfilled all the requirements ... I highly recommend hiring her.

Sincerely,
Julius Caesar

colliding order

... full access to all confidential and secret information ...

Sincerely,
Julius Caesar

\[ \text{MD5} \]

\[ 5421a523481f6dc6a2a1c832e72c7b8a5 \]

Image Credit: Lucks and Daum, Eurocrypt 2005
Modular differential attack

We want to find a pair \((M_0, M_1)\) and \((M_0', M_1')\) such that:
\[
(a, b, c, d) = \text{md5}(a_0, b_0, c_0, d_0, M_0),
\]
\[
(a', b', c', d') = \text{md5}(a_0, b_0, c_0, d_0, M_0'),
\]
\[
\text{md5}(a, b, c, d, M_1) = \text{md5}(a', b', c', d', M_1'),
\]

- Use modular integer subtraction as the measure of difference
  - Finding the first blocks \((M_0, M_0')\) takes about \(2^{39}\) md5 operations
  - Finding the second blocks \((M_1, M_1')\) takes about \(2^{32}\) md5 operations
Finding useful collisions

Lucks and Daumen, Eurocrypt 2005

1. Use advanced document language, e.g., postscript
2. Find random strings $R_1$ and $R_2$ and concatenate to some preamble:

$$X_1 = \text{preamble}; \text{put}(R_1);$$
$$X_2 = \text{preamble}; \text{put}(R_2);$$

$$\text{md5}(X_1) = \text{md5}(X_2)$$

Append a string $S$ to both $X_1$ and $X_2$:

$$\text{md5}(X_1 \parallel S) = \text{md5}(X_2 \parallel S)$$

More info: http://th.informatik.uni-mannheim.de/people/lucks/HashCollisions/
Finding useful collisions

Target documents are $T_1$ and $T_2$: 

\[
Y_1 = \text{preamble}; \text{put}(R_1); \text{put}(R_1); \text{if}(=) \text{ then } T_1 \text{ else } T_2
\]

\[
Y_2 = \text{preamble}; \text{put}(R_2); \text{put}(R_1); \text{if}(=) \text{ then } T_1 \text{ else } T_2
\]

Viewing $Y_1$: $R_1 = R_1$, thus $T_1$ is displayed.

Viewing $Y_2$: $R_2 \neq R_1$, thus $T_2$ is displayed.
May, 22, 2005

To Whom it May Concern:

Alice Falbala fulfilled all the requirements of the Roman Empire intern position. She was excellent at translating roman into her gaul native language, learned very rapidly, and worked with considerable independence and confidence.

Her basic work habits such as punctuality, interpersonal deportment, communication skills, and completing assigned and self-determined goals were all excellent.

I recommend Alice for challenging positions in which creativity, reliability, and language skills are required.

I highly recommend hiring her. If you’d like to discuss her attributes in more detail, please don’t hesitate to contact me.

Sincerely,

Julius Caesar
SHA-256

- If you need a one-way hash function, this is the one to use
- 256-bit SHA-2 hash function designed by the NSA

Should be used as checksum for downloads:

```bash
$ sha256sum ubuntu-14.04.3-desktop-amd64.iso
756a42474bc437f614caa09dbbc0808038d1a586d172894c1
13bb1c22b75d580  ubuntu-14.04.3-desktop-amd64.iso
```

- Input: 512-bit blocks, divided into 32-bit words
- Output: After 64 rounds, the final result is the sum, by individual words modulo $2^{32}$, of the result of this transformation and the original eight word input
One SHA-2 Round

$W_t = t^{th}$ 32-bit word of the message schedule

$K_t = constant$
Operations performed in each round

\[ \text{Ch} (E, F, G) = (E \land F) \oplus (\neg E \land G) \]
\[ \text{Ma} (A, B, C) = (A \land B) \oplus (A \land C) \oplus (C \land C) \]
\[ \sum_0 (A) = (A \gg 2) \oplus (A \gg 13) \oplus (A \gg 22) \]
\[ \sum_1 (E) = (E \gg 6) \oplus (E \gg 11) \oplus (E \gg 25) \]
void sha256(char *string, char outputBuffer[65])
{
    unsigned char hash[SHA256_DIGEST_LENGTH];
    SHA256_CTX sha256;
    SHA256_Init(&sha256);
    SHA256_Update(&sha256, string, strlen(string));
    SHA256_Final(hash, &sha256);
    int i;
    for(i = 0; i < SHA256_DIGEST_LENGTH; i++)
    {
        sprintf(outputBuffer + (i * 2), "%02x", hash[i]);
    }
    outputBuffer[64] = 0;
}
Symmetric Key Cryptography

How do two actors communicate securely?

1. Alice and Bob agree on a cryptosystem and key

2. Alice encrypts her plaintext message using the chosen cryptosystem and key. This creates a ciphertext message.
Symmetric Key Cryptography

3. Alice sends the ciphertext message to Bob.

\[ \text{Alice} \xrightarrow{\text{...}} \{X\}_{k_{AB}} \xrightarrow{\text{...}} \text{Bob} \]

4. Bob decrypts the cipher text message with the same cryptosystem and key

\[ \{X\}_{k_{AB},k_{AB}} \xrightarrow{\text{...}} X \]
Stream Ciphers

Some encryption algorithms use a non-repeating stream of key elements to encipher characters of a message.
Stream Ciphers

Definition: Let $E$ be an encryption algorithm, and let $E_k(X)$ be the encryption of message $X$ with key $k$. Let a message $X = x_1x_2\ldots$, where each $x_i$ is of a fixed length, and let $k = k_1k_2\ldots$. Then a stream cipher is a cipher for which $E_k(X) = E_{k_1}(x_1)E_{k_2}(x_2)\ldots$.

If the key stream $k$ of a stream cipher repeats itself, it is a periodic cipher.
Synchronous Stream Ciphers

- Goal: simulate a random, infinitely long key
  - Extract bits from a register to use as the key
    - Contents of the register change on the basis of the current contents of the register

Period of 8:
10110001 10110001 10110001...
Linear Feedback Shift Register

Definition: An *n*-stage linear feedback shift register (LFSR) consists of an *n*-bit register \( r = r_0 \ldots r_{n-1} \) and an *n*-bit *tap sequence* \( t = t_0 \ldots t_{n-1} \). To obtain a key bit, \( r_{n-1} \) is used, the register is shifted one bit to the right, and the new bit \( r_0 t_0 \oplus \ldots \oplus r_{n-1} t_{n-1} \) is inserted.
**Linear Feedback Shift Register Example**

Tap sequence: 10 01

<table>
<thead>
<tr>
<th>Current register</th>
<th>Key</th>
<th>New bit</th>
<th>New register</th>
</tr>
</thead>
<tbody>
<tr>
<td>00 10</td>
<td>0</td>
<td>01⊕00⊕10⊕01 = 0⊕0⊕0⊕0 = 0</td>
<td>00 01</td>
</tr>
<tr>
<td>00 01</td>
<td>1</td>
<td>01⊕00⊕00⊕11 = 0⊕0⊕0⊕1 = 1</td>
<td>10 00</td>
</tr>
<tr>
<td>10 00</td>
<td>0</td>
<td>11⊕00⊕00⊕01 = 1⊕0⊕0⊕0 = 1</td>
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</tr>
<tr>
<td>11 11</td>
<td>1</td>
<td>11⊕10⊕10⊕11 = 1⊕0⊕0⊕1 = 0</td>
<td>01 11</td>
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</table>

Period of 15, key is 010001111010110
Self-synchronous Stream Ciphers

Some stream ciphers obtain the key from the message itself. Two strategies for this:

1. Draw the key from the plaintext (Vigenère cipher from book):
   - key: XTHEBOYHASTHEBA
   - plaintext: THEBOYHASTHEBAG
   - ciphertext: QALFPNFHSLALFCT

2. Draw the key from the ciphertext:
   - key: XQXBCQOVVNGNRTT
   - plaintext: THEBOYHASTHEBAG
   - ciphertext: QXBCQOVVNGNRTTM