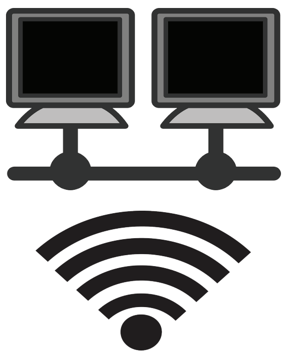
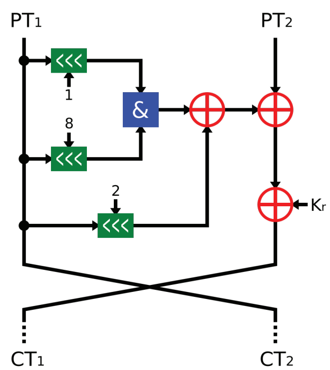
CSE 40567 / 60567: Computer Security





Cryptography 4

Homework #2 has been released. It is due Thursday, Feb. 6th at 11:59PM

See **Assignments Page** on the course website for details

Advanced Encryption Standard (AES)

- If you need a symmetric key algorithm, this is the one to use
- Based on Rijndael, winner of the 2001 NIST AES competition
- Key sizes: 128-, 192- or 256-bit
- Block size: 128-bit
- Rounds: 10, 12 or 14 (depending on key size)

FIPS PUB 197: Advanced Encryption Standard (AES)

ISO/IEC 18033-3: Information technology — Security techniques — Encryption algorithms — Part 3: Block ciphers

AES is a substitution-permutation network

- Works via a combination of both substitution and permutation operations
    - Fast in both hardware and software
- Operates on 4x4 column-major order matrix of bytes (state)

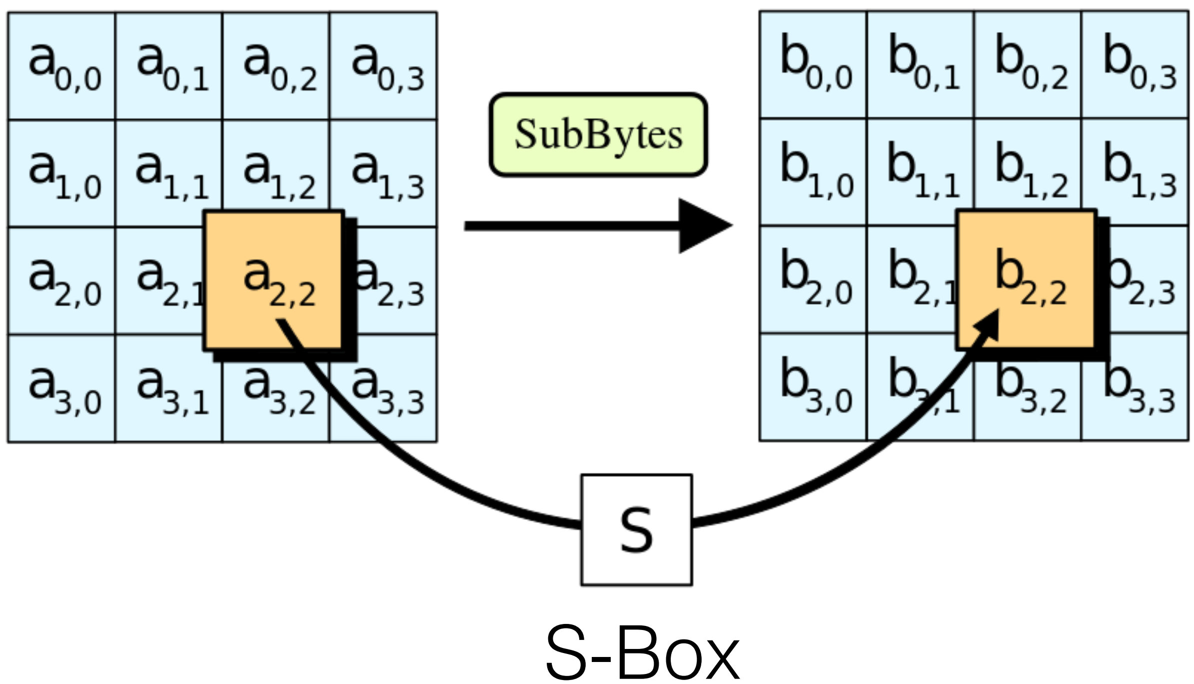
$$\begin{bmatrix} b_0 & b_4 & b_8 & b_{12} \\ b_1 & b_5 & b_9 & b_{13} \\ b_2 & b_6 & b_{10} & b_{14} \\ b_3 & b_7 & b_{11} & b_{15} \end{bmatrix}$$

High-level overview of AES

1. **KeyExpansions —** round keys are derived from k using a key schedule
2. **InitialRound**
    1. AddRoundKey
3. **Rounds**
    1. SubBytes
    2. ShiftRows
    3. MixColumns
    4. AddRoundKey
4. **Final Round (no MixColumns)**
    1. SubBytes
    2. ShiftRows
    3. AddRoundKey

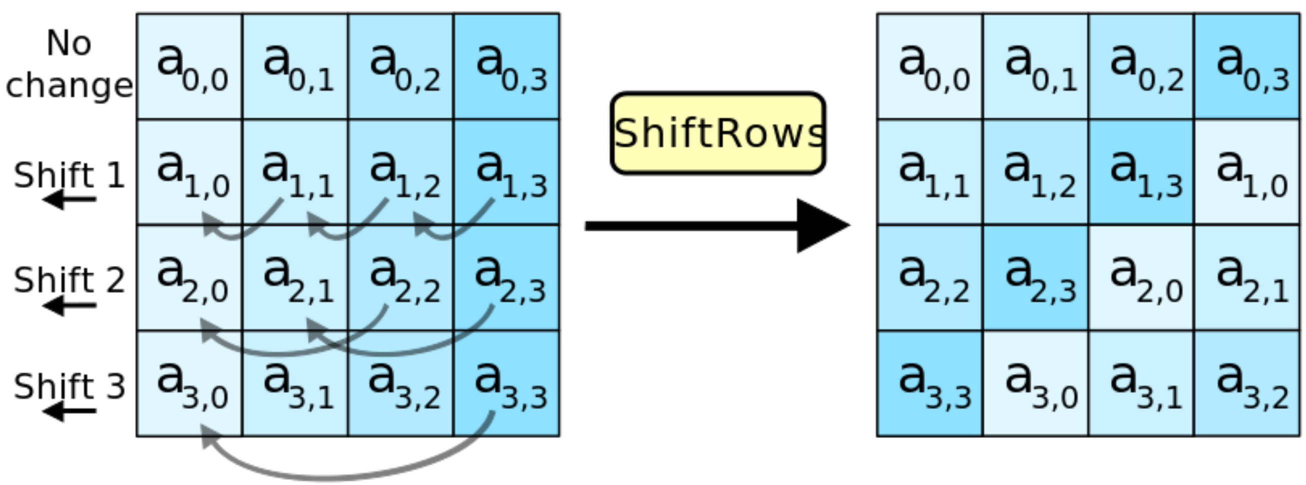
SubBytes Step

Each byte in the state is replaced with its entry in a fixed 8-bit lookup table (a substitution box), S; $b_{i,j} = S(a_{i,j})$.



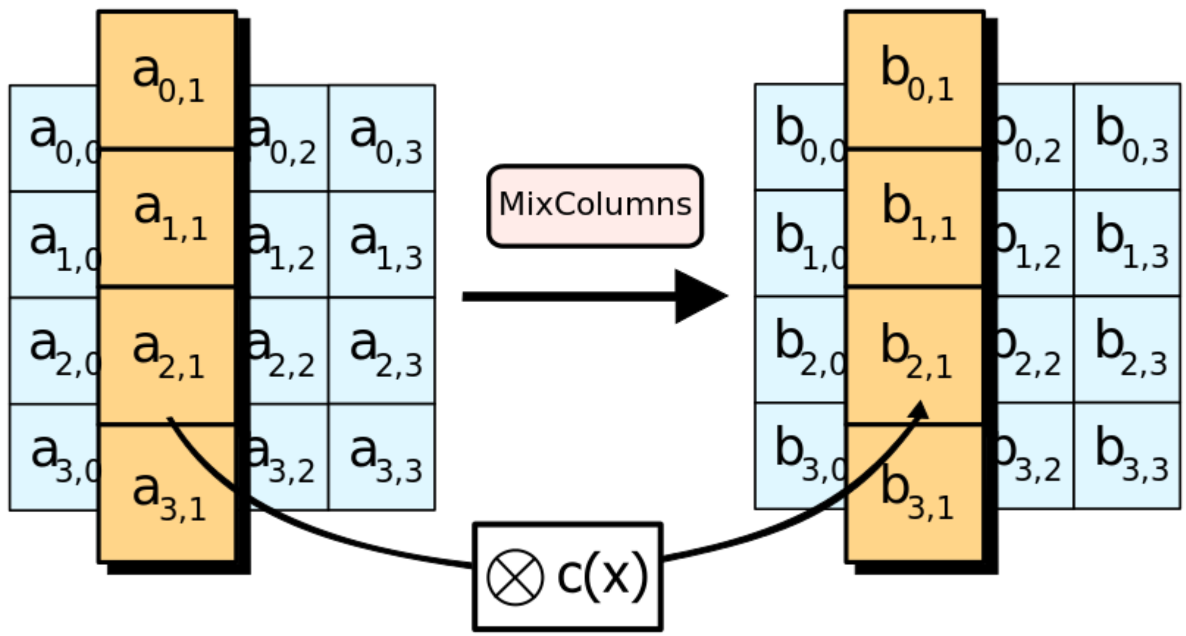
ShiftRows Step

Bytes in each row of the state are shifted cyclically to the left. The number of places each byte is shifted differs for each row.



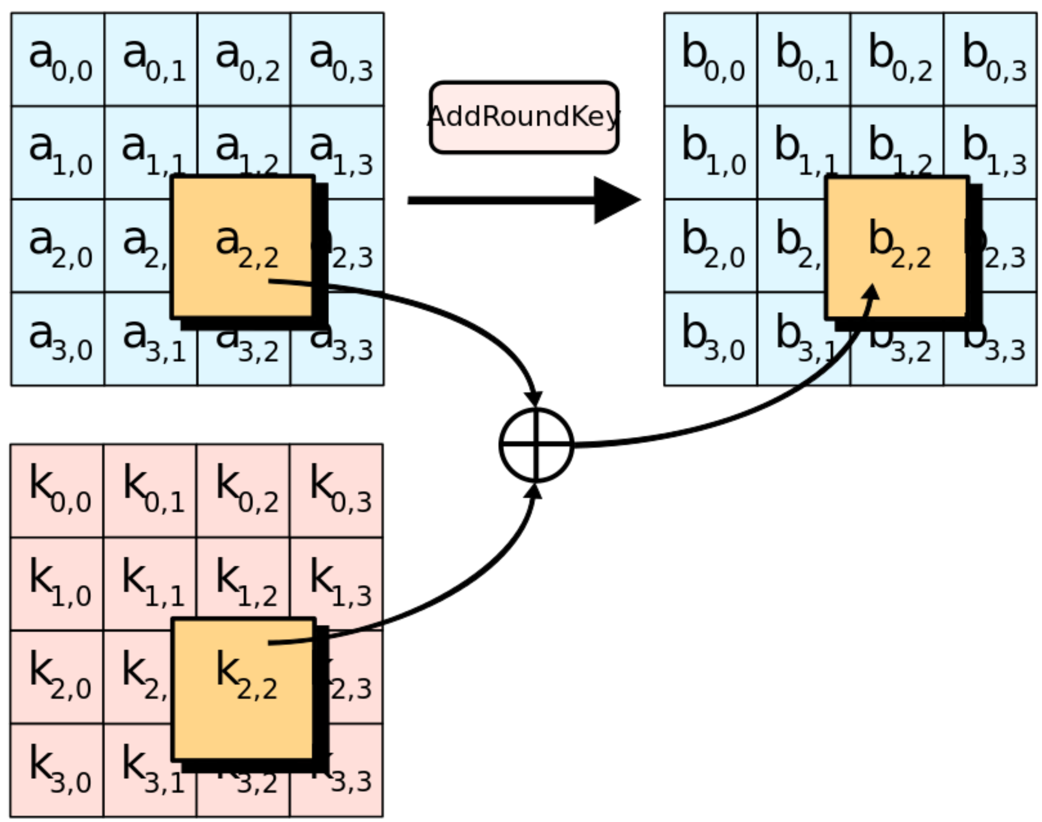
MixColumns Step

Each column of the state is multiplied with a fixed polynomial $c(x)$.



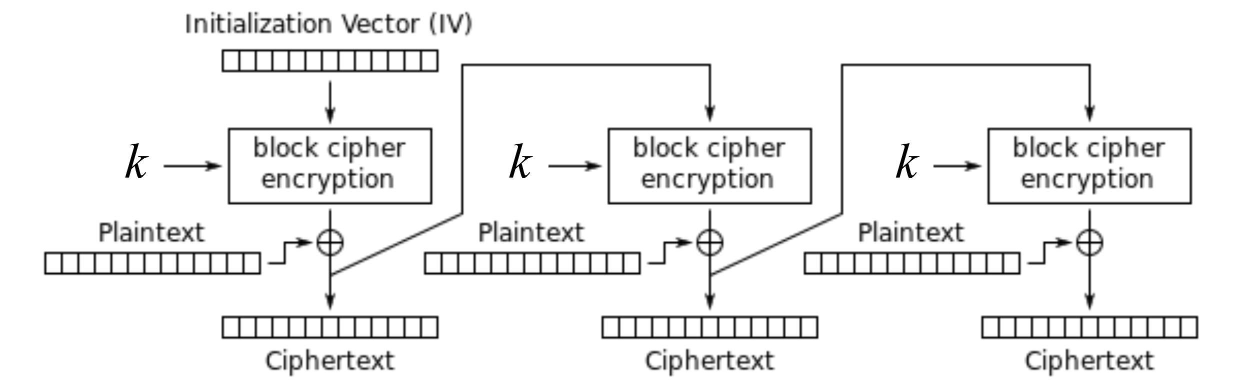
AddRoundKey Step

Each byte of the state is XORed with a byte of the round subkey.

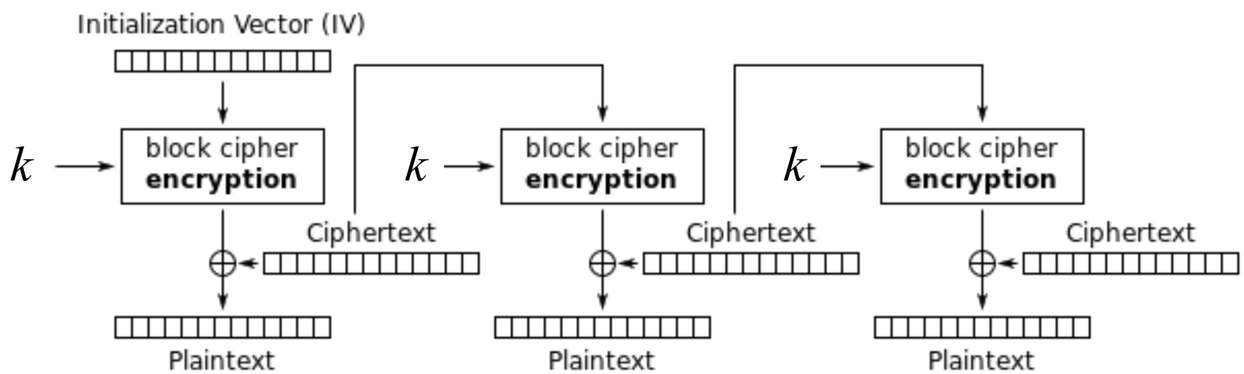


AES Compatible Stream Modes

Cipher Feedback (CFB) Mode, a variation of CBC that turns a block cipher into a self-synchronizing stream cipher



Cipher Feedback (CFB) mode encryption

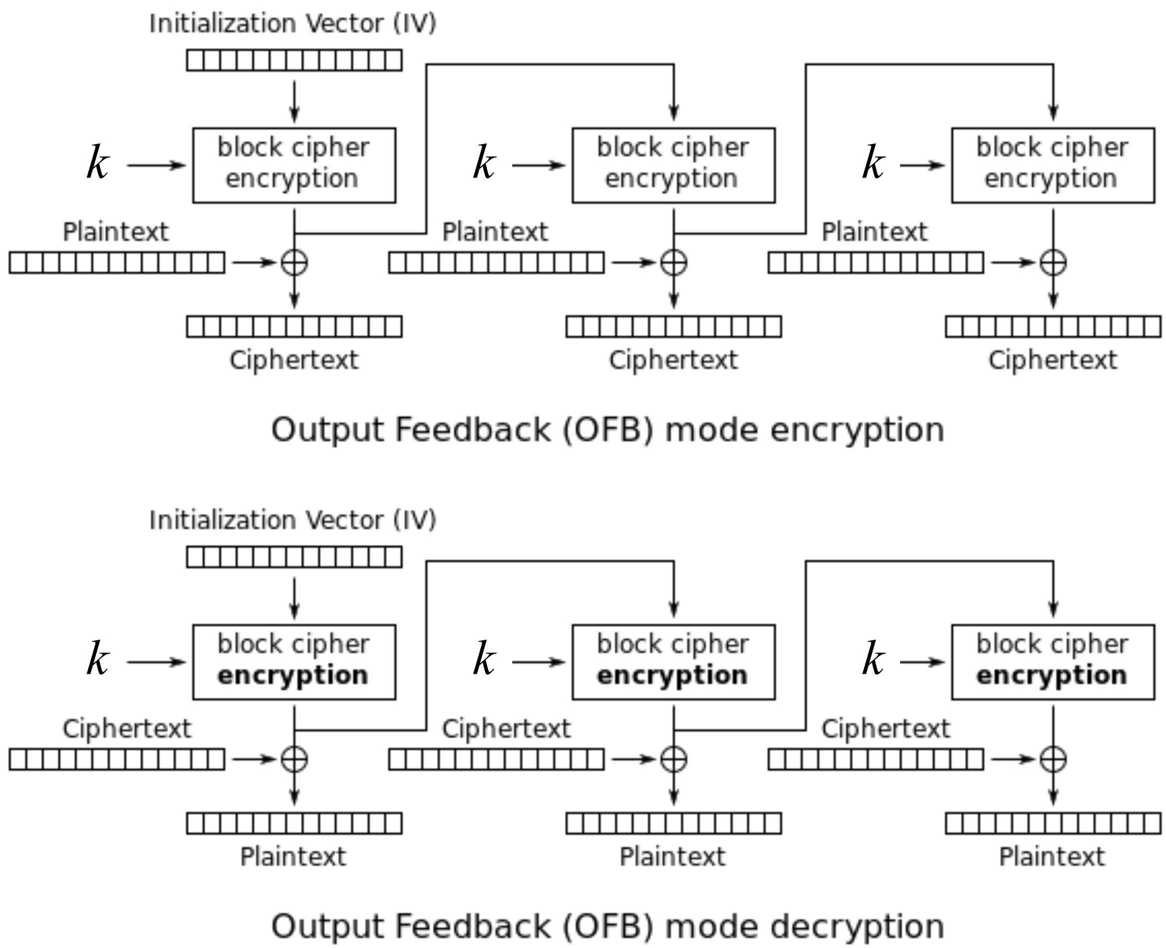


Cipher Feedback (CFB) mode decryption

Downside: **Slow**

AES Compatible Stream Modes

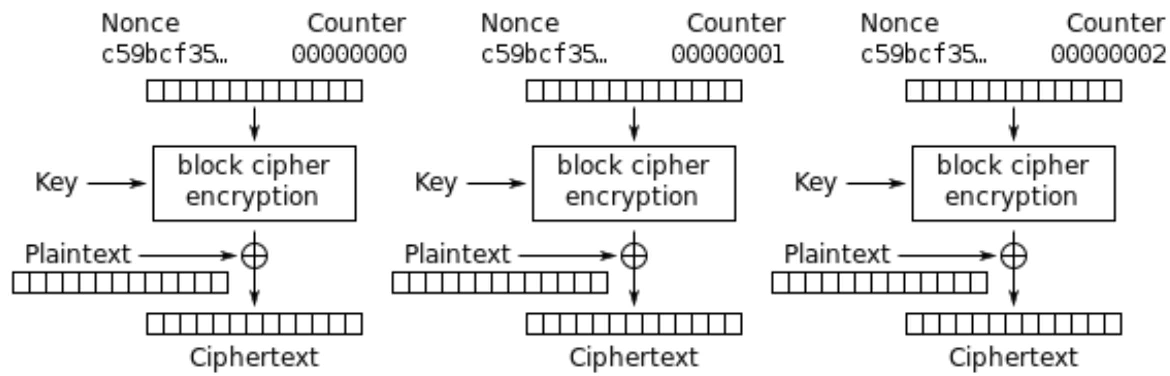
Output Feedback (OFB) Mode also turns a block cipher into a self-synchronizing stream cipher



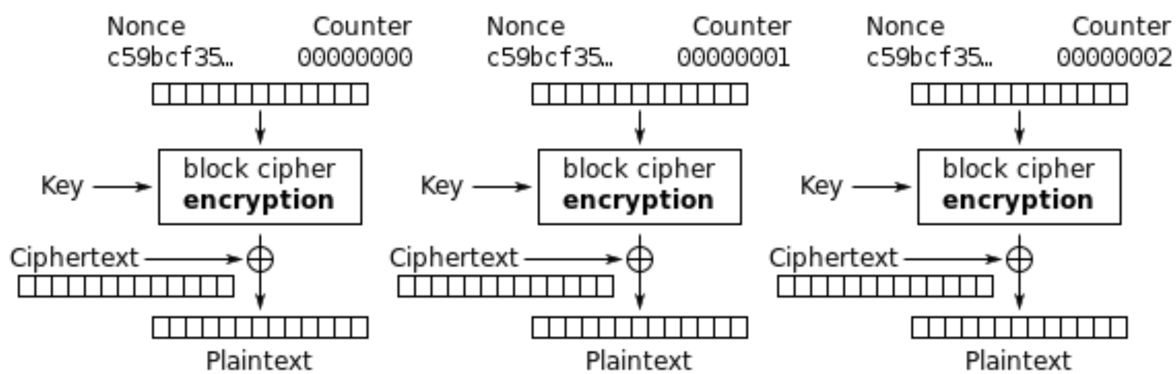
Downside: **Slow**

AES Compatible Stream Modes

Counter (CTR) Mode generates the next keystream block by encrypting successive values of a counter combined with a nonce (IV)



Counter (CTR) mode encryption



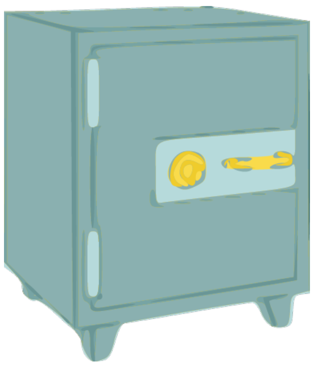
Counter (CTR) mode decryption

*Use this mode when a stream cipher is needed

AES functions in LibreSSL / OpenSSL

```
#include <openssl/aes.h>

int AES_set_encrypt_key(const unsigned char *userKey, const int bits,
        AES_KEY *key);
int AES_set_decrypt_key(const unsigned char *userKey, const int bits,
        AES_KEY *key);

int private_AES_set_encrypt_key(const unsigned char *userKey, const int bits,
        AES_KEY *key);
int private_AES_set_decrypt_key(const unsigned char *userKey, const int bits,
        AES_KEY *key);

void AES_cbc_encrypt(const unsigned char *in, unsigned char *out,
        size_t length, const AES_KEY *key,
        unsigned char *ivec, const int enc);

void AES_ctr128_encrypt(const unsigned char *in, unsigned char *out,
        size_t length, const AES_KEY *key,
        unsigned char ivec[AES_BLOCK_SIZE],
        unsigned char ecount_buf[AES_BLOCK_SIZE],
        unsigned int *num);
```

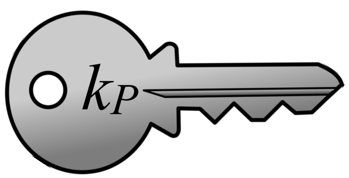
Public Key Cryptography



Clip art of a safe

- A symmetric algorithm is like a safe
  - The key is the combo
  - Anyone with the combo can open the safe
  - Anyone without the combo must learn safecracking

1976: Whitfield Diffie and Martin Hellman introduce alternative paradigm with two keys:

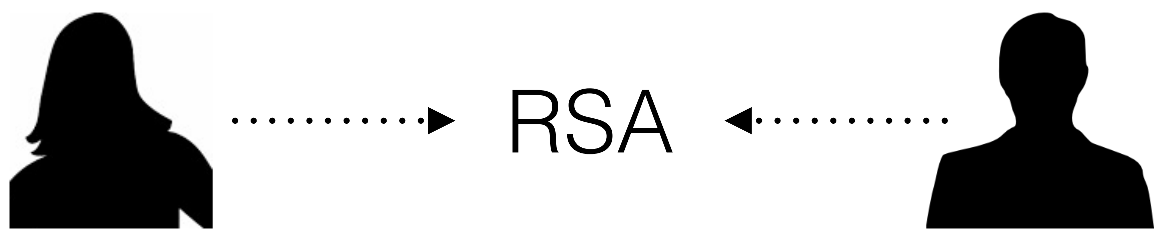


Public (sharable)

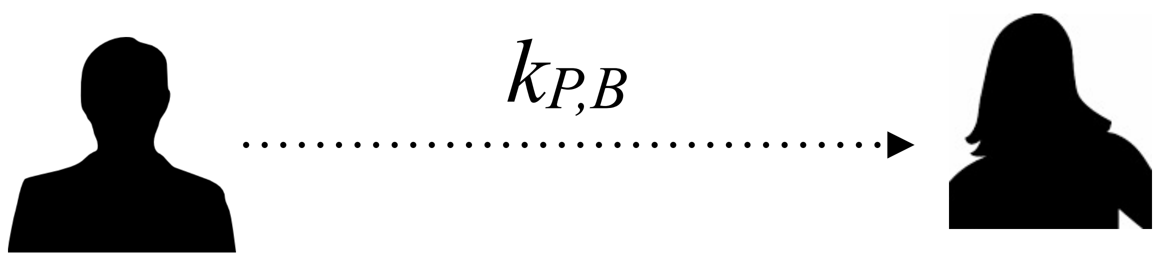
Private (secret)

Sending a message using public key crypto

1. Alice and Bob agree on a public key crypto system

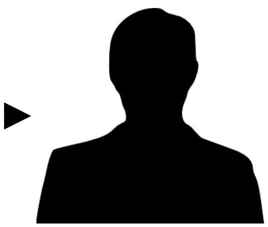


2. Bob sends Alice his public key



Sending a message using public key crypto

3. Alice encrypts her message using Bob's public key and sends it back to Bob

$X,k_{P,B} \longrightarrow$  $\xrightarrow{\{X\}_{k_{P,B}}}$ 

4. Bob decrypts Alice's message using his private key

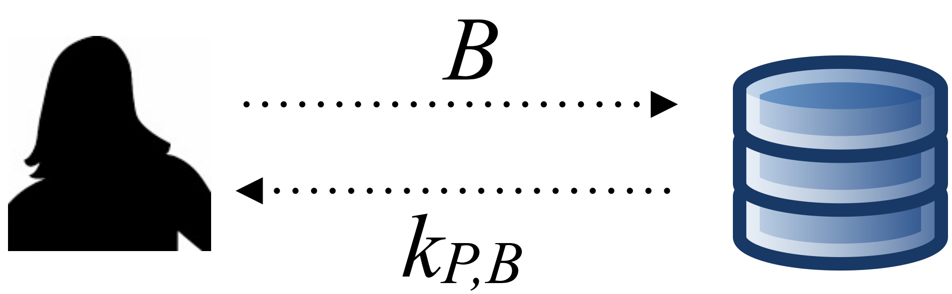
 $\{X\}_{k_{P,B}}, k_{S,B} \longrightarrow X$

Sending a message using public-key crypto and a central key repository

- The protocol we just saw is clunky: Alice needs to contact Bob before sending him a message

- If public keys are stored in an accessible database, the protocol is simplified to three steps:

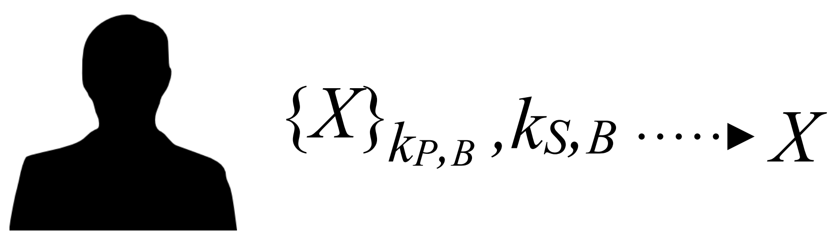
1. Alice gets Bob's key from the database



Sending a message using public-key crypto and a central key repository

2. Alice encrypts her message using Bob’s public key and sends it back to Bob

3. Bob decrypts Alice’s message using his private key



Bob isn’t involved until he reads his message

Practical public key crypto application: email encryption via GPG


```
-----BEGIN PGP PUBLIC KEY BLOCK-----
Version: GnuPG v1.4.9 (GNU/Linux)

mQGiBEKD9XwRBAC0h4q4ueWr0skKkTDP+XKwZ6HSTYFkfxjAUlITyZpGBajYZuXO
y/u7HTAkggS/eSX0VErNFrIUIkdXh0ED/FQxE7tq5tBQv22i/ehoakeww9RNYy0e
yUSYrpCFh4Ktszo2LXIAD5HfEmKI8ow6pHT8PKBn2oqZyYo/nFvTzG7GlwCghzKt
mUJ6dds7ONZvpGbMBeI/0JsEAIY7dPb9lM1Xzauk1o9cmKrnL2P7SK8vsQZUvcZL
7X7dI4J0TWYK19ShOXjQNA7CJbbvlOuKJuXwLsH/VzX+MD3P9l0ZbDEQeAtu4mLG
kxcYMU/rPOjuC7erPDmL47+N/sS/2qHO21lKf2SuIkry57ikcZxdt5czLztWFc9l
fXwF3ICIbZHA3i8An1B9dLXBHfNNFajQIZrdwfw+gDktiEYEEBECAAYFAkZ3/9oA
CgkQlWQfayU+WOOO5QCgqCrojF3nDPhcwGK+Ft0v9UmivRAAoK5cOokgf35eF034
LXOYpe0eT0omP8CJBQzqvN0jtUZ94Vux6tgV8eygEOKlQibSYodQSHTKq+WKKXIt
dGy+/kmj1LE9py8vkfioHOAFhHfVJyx8DuEXIzBnFwXr8E822hqN/qt5Mq9y9OBy
MrFaOfZ7YdcV1y/yYooTvQA78A3gyFle7vBsEJC7xo4eTdt2/9/kiSFZ3mGAsJKe
4dB61rhlCa5gtVQH0Z/HRRNmUC1PC8Wph/u2z8sgT6BYf59mX4q8gi60Ar30g3IF
XEWhWIhJBBgRAgAJBQJCg/W5AhsMAAoJEAMkDQZT2UAUWYMAn2NBNHIOJMcnj8Oo
FIgyxFGXBslCAJ4zcUz74RbQuP+UV/hPf2OlY7SeOA==
=q2SM
-----END PGP PUBLIC KEY BLOCK-----
```


Image Credit: http://www.oekonux-conference.org/documentation/texts/Meyer.html

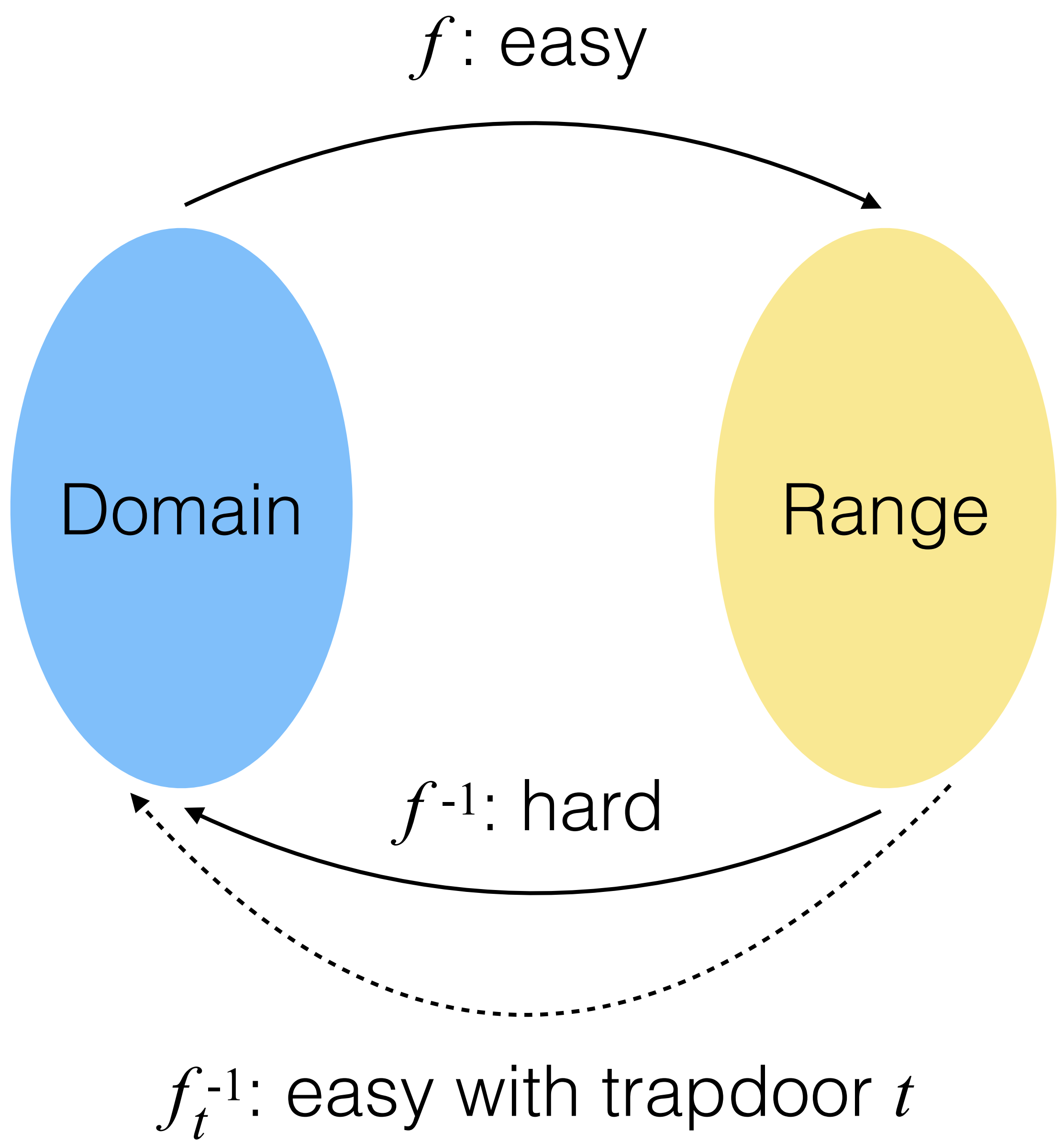
- Free implementation of OpenPGP standard (RFC4880)
- Public key encryption and signing of data and communications
- Versatile key mangement system

```
# apt-get install gnupg
```

How does public key crypto work?

- How do we generate two keys that work together?
- How do we make sure the public key doesn't reveal any information about the private key?
- How can we design the algorithm to be resilient to chosen plaintext attacks: an attacker can choose any message to encrypt

Trapdoor Function



RSA

- First full-fledged public key encryption algorithm
    - 1978: Ron Rivest, Adi Shamir, and Leonard Adleman
- If you need a public key encryption algorithm, this is one to use
- Simple to understand and implement
    - **Security comes from the difficulty of factoring large numbers**

R. L. Rivest, A. Shamir, and L.M. Adleman, “A Method for Obtaining Digital Signatures and Public-Key Cryptosystems,” Communications of the ACM, vol. 21, no. 2, 1978

R. L. Rivest, A. Shamir, and L.M. Adleman, “On Digital Signatures and Public Key Cryptosystems,” MIT Laboratory for Computer Science, Technical Report, MIT/LCS/TR-212, 1979

RSA Key Generation

1. Choose two random large prime numbers of equal length, p and q.

2. Compute the product (modulus): $n = p \cdot q$

3. Randomly choose the encryption key e such that e and $(p - 1)(q - 1)$ are relatively prime

4. Use the extended Euclidean algorithm to compute the decryption key, d, such that:

$$ed \equiv 1 \bmod (p - 1)(q - 1)$$

$$d = e^{-1} \bmod ((p - 1)(q - 1))$$

RSA Key Generation

- d and n are also relatively prime
- The numbers e and n are the public key k_P
- The number d is the private key k_S
- The two primes, p and q, are no longer needed. They must be discarded and never revealed.

RSA Encryption

1. Divide a message X into numerical blocks x_i that are smaller than n (with binary data, choose the largest power of 2 less than n).

   The encrypted messages, $\{X\}$, will be made up of similarly sized message blocks $\{x_i\}$, of the same length

2. Apply the encryption formula:

$$\{x_i\} = x_i^e \bmod n$$

B. Schneier, Applied Cryptography, Chpt. 19

RSA Decryption

1. Take each encrypted block $\{x_i\}$ and apply the decryption formula:

$$x_i = \{x_i\}^d \bmod n$$

The message could have also been encrypted with d and decrypted with e. The use of the keys is arbitrary!

A short example

Key Generation:

1. $p = 47$ and $q = 71$

2. $n = p \cdot q = 3337$

3. The encryption key e must have no factors in common with: $(p - 1)(q - 1) = 46 \cdot 70 = 3220$

4. Choose e at random to be 79. Calculate d:
$d = 79^{-1} \bmod 3220 = 1019$ ← solved via extended Euclidean Alg.

5. Publish e and n, keep d secret. Discard p and q.

A short example

Encryption:

1. $X = 6882326879666683$

2. Break X into small blocks (3 digits here):

$$\begin{aligned} x_1 &= 688 \\ x_2 &= 232 \\ x_3 &= 687 \\ x_4 &= 966 \\ x_5 &= 668 \\ x_6 &= \boxed{00}3 \end{aligned}$$

3. The first block is encrypted as: $688^{79} \bmod 3337 = 1570 = \{x_1\}$

4. Repeating the encryption operation on subsequent blocks yields: $\{X\} = 1570\ 2756\ 2091\ 2276\ 2423\ 158$

A short example

Decryption:

1. First block: $1570^{1019} \bmod 3337 = 688 = x_1$

2. Subsequent blocks are recovered in the same manner

How do we generate prime numbers?

- If we always need different prime numbers, won't we run out?
  - No: there are approximately 10^{151} primes 512 bits in length or less (by comparison, there are only 10^{77} atoms in the universe)
- What if two people accidentally pick the same prime number?
  - Improbable (you have much better odds at the Powerball)
- What if somebody creates a database of all primes?
  - Impossible (would exceed physical limits of the universe)

How do we generate prime numbers?

1. Select a random number of a desired length
2. Apply a **Fermat primality test** (best with base 2 for speed optimization)
3. Apply a certain number of **Miller-Rabin primality tests** (depending on the length and allowed error rate)

Pre-selection: test divisions by small prime numbers (up to few hundreds) or sieve out primes up to 10,000 - 1,000,000 considering many prime candidates of the form $b + 2i$

large number   up to a few thousands

Speed of RSA

Slow — use to transfer symmetric session keys for the bulk of the encryption

3.1 Ghz Intel Core i7

The numbers are in 1000s of bytes per second processed.

```
$ openssl speed rsa
                  sign    verify    sign/s verify/s
rsa  512 bits 0.000134s 0.000010s   7437.2 103127.2
rsa 1024 bits 0.000454s 0.000022s   2203.4  45325.5
rsa 2048 bits 0.002358s 0.000063s    424.0  15980.3
rsa 4096 bits 0.014172s 0.000200s     70.6   4993.3
```

```
$ openssl speed aes
type             16 bytes     64 bytes    256 bytes   1024 bytes   8192 bytes
aes-128 cbc     145688.81k   149145.51k   150745.15k   147818.26k   150304.14k
aes-192 cbc     127548.47k   128854.67k   130738.82k   130399.58k   129314.59k
aes-256 cbc     111384.28k   107228.01k   111353.87k   113593.73k   116542.79k
```

RSA key sizes

"Attacks always get better; they never get worse."

-NSA Aphorism (as related by Bruce Schneier)

What is considered to be secure in 2019?

Research suggests 1024-bit moduli are too small
(i.e., NSA can factor them):

https://weakdh.org/imperfect-forward-secrecy-ccs15.pdf

If you are protecting 128-bit AES keys, 2,048-bit moduli are adequate

RSA functions in LibreSSL / OpenSSL

```
#include <openssl/rsa.h>

RSA * RSA_new(void);
void RSA_free(RSA *rsa);

int RSA_public_encrypt(int flen, unsigned char *from,
    unsigned char *to, RSA *rsa, int padding);
int RSA_private_decrypt(int flen, unsigned char *from,
    unsigned char *to, RSA *rsa, int padding);
int RSA_private_encrypt(int flen, unsigned char *from,
    unsigned char *to, RSA *rsa,int padding);
int RSA_public_decrypt(int flen, unsigned char *from,
    unsigned char *to, RSA *rsa,int padding);

RSA *RSA_generate_key(int num, unsigned long e,
    void (*callback)(int,int,void *), void *cb_arg);

int RSA_check_key(RSA *rsa);
```